

Novel coupling applicable in off-centered condition

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Abstract

A novel coupling was devised by utilizing the self-locking property of a belt. The torque is transmitted through a flexible belt. The belt length between the power disk and the driven shaft varies with a rotation of shaft in an off-centered condition. It causes an angular velocity fluctuation when the torque is transmitted by a single belt. In order to decrease this fluctuation in angular velocity, the looped belt method was devised. The looped belt is formed with the aid of the cam-followers to coil around the driven shaft so as to transmit torque through two points opposed. By running a looped belt so as to cancel the change in belt length between the power disk to the driven shaft, the fluctuation of angular velocity was successfully suppressed. The theory of looped belt coupling was examined. Experiments were carried out to confirm the theory of the looped belt method. Simple fatigue tests were also carried out to evaluate durability of the coupling. These tests showed that the looped belt method was quite effective in reducing angular velocity fluctuation in transmitting torque in an off-centered condition. It also has practical durability.

Key words : Coupling, Shaft misalignments, Looped belt, Belt friction, Self-lock, Angular velocity

1. Introduction

Coupling is a machine component which connects the output shaft of a motor to the input shaft of various driven units. These units have to be carefully aligned so that they can be connected to one another. This is necessary so that unfavorable vibration does not occur that may damage the machine unit. However, achieving accurate alignment of the shafts is often difficult, so specific couplings are necessary that allow for small misalignments of the shafts. There are three kinds of shaft misalignments: parallel misalignment, in-line misalignment and angular misalignment (Orthwein., 1999). All these misalignments are usually present. A suitable coupling is selected depending on the situation that is expected. However, in general, only a small amount of misalignment is permitted. For large parallel misalignment, either Oldham's coupling or Schmidt coupling is used. However, these couplings accommodate very small tolerance for angular and in-line misalignment and require ample lubrication (Orthwein., 1999).

Imado found the self-locking property of the belt in the course of his study and showed the theoretical self-locking condition for two and three times wrapping condition of belt (Imado, 2008; Imado, et al., 2013a). He devised a one-way clutch by utilizing the self-locking property of belt (Imado, et al., 2010; Imado, et al., 2013b). This clutch can transmit torque even under some off-centered conditions. Because the torque is transmitted through a flexible belt, shaft alignment is much less critical than for conventional one-way clutches. However, the ratio of angular velocities fluctuates according to the magnitude of an off-centered distance. The fluctuation in angular velocity was caused by the change in the belt length between the power ring and the inner shaft of the clutch. Paying careful attention to this fact, the fluctuation of angular velocity was analyzed and compared with some experimental values in the previous paper (Imado, et al., 2013b). In terms of practical use, the fluctuation in angular velocity is undesirable. This study was carried out to develop an effective method for reducing the fluctuation in angular velocity. We devised a looped-belt method to reduce the fluctuation of angular velocity. The paper discusses the principle of the looped belt coupling. Geometrical consideration was made to explain the experimental results. Simple fatigue tests were carried out to check the durability of the looped belt coupling. It was confirmed that the looped belt method was very effective in reducing

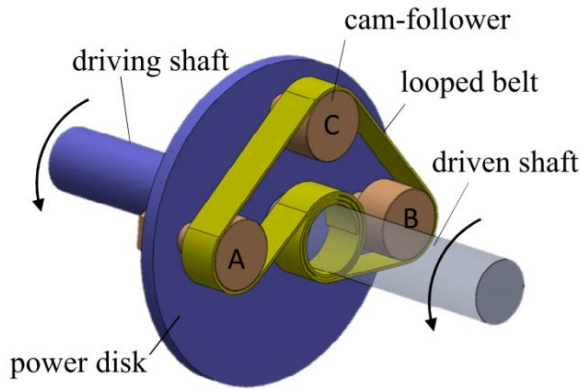


Fig. 1 Three dimensional image of looped belt coupling
Rotational power is transmitted to the driven shaft by self-locked looped belt.

angular velocity fluctuation in transmitting torque in an off-centered condition. It also has practical durability.

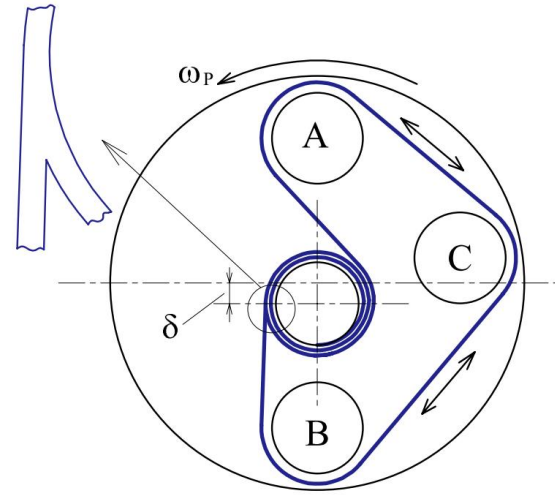


Fig. 2 Front view image of the looped belt coupling

2. Principle of the looped belt coupling

Figure 1 shows the 3D image of the looped belt coupling. Three cam-followers A, B and C are mounted on the power disk. The looped belt is wound around the driven shaft through cam-followers A, B and C. The main role of the cam-follower C is to prevent the belt from coming in contact with the driven shaft. The characteristic of this coupling is that the rotational torque is transmitted through the two opposing belts. In the case of an off-centered condition, the length of an each belt changes sinusoidally with the rotation of the power disk. Therefore, the two opposing belts change in a way as one belt becomes longer, the other belt becomes shorter so as to cancel the total change in the belt length during the rotation of the shaft. Because an each opposing belt is a part of the looped belt, the belt moves to the larger belt tension side to equilibrate its tension. In the case of an off-centered condition, the looped belt moves right and left on the cam-followers to balance its tension. These mechanisms work effectively to reduce fluctuation in angular velocity with an off-centered condition. Figure 2 is a front view image of the looped belt coupling to illustrate a looped belt and the definition of an off-centered distance δ .

3. Nomenclature

a	: radius of revolution of the cam-follower, m
b_i	: length of belt between contact boundaries ($i=1, 2, 3, 4$), m
r	: radius of cam-follower, m
t	: time, s
L_i	: length of belt ($i=1$ and 2), m
L_{mean}	: mean length of belt, i.e. $(L_1+L_2)/2$
P_i	: contact boundary ($i=1, 2, \dots, 8$)
R	: radius of the driven shaft, m
T_b	: brake torque, Nm
δ	: magnitude of off-centered distance, m
θ_A	: angle of revolution of the cam-follower A
θ_B	: angle of revolution of the cam-follower B
θ_P	: angle of revolution of the power disk
ϕ_i	: angle ($i=2, 3, \dots, 7$)
ω	: angular velocity of the driven shaft, rad/s
ω_P	: angular velocity of the power disk, rad/s

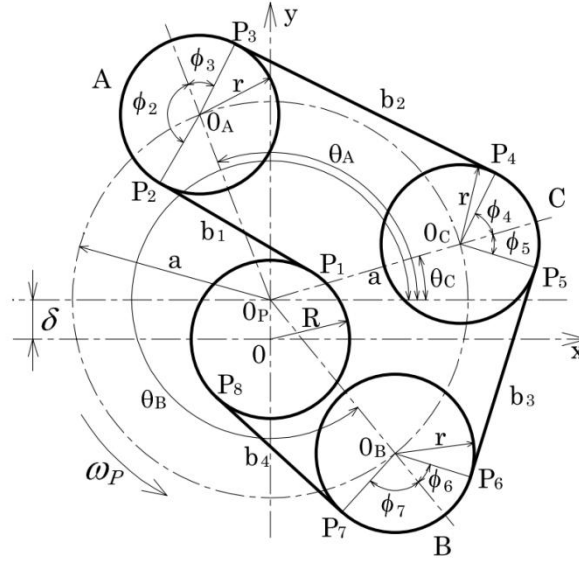


Fig. 3 Mechanical model of the looped belt coupling

4. Theory

Figure 3 is a mechanical model of the looped belt coupling. For the sake of simplicity, the thickness of the belt is not considered. O_P denotes the center of power disk and O denotes that of the driven shaft. O_A , O_B and O_C denote centers of cam-followers. Let us consider a closed path that consists of points O , O_P , O_A , P_2 , P_1 and O . Each segment is defined by the following vectors:

$$\left. \begin{aligned} \overrightarrow{OO_P} &= \begin{pmatrix} 0 \\ \delta \end{pmatrix}, & \overrightarrow{O_P O_A} &= a \begin{pmatrix} \cos \theta_A \\ \sin \theta_A \end{pmatrix} \\ \overrightarrow{O_A P_2} &= r \begin{pmatrix} \cos(\theta_A + \phi_2) \\ \sin(\theta_A + \phi_2) \end{pmatrix}, & \overrightarrow{P_2 P_1} &= b_1 \begin{pmatrix} -\sin(\theta_A + \phi_2) \\ \cos(\theta_A + \phi_2) \end{pmatrix} \end{aligned} \right\} \quad (1)$$

Note that $P_1 O$ is parallel to the $O_A P_2$, Eq. (2) is also derived.

$$\overrightarrow{P_1 O} = R \begin{pmatrix} \cos(\theta_A + \phi_2) \\ \sin(\theta_A + \phi_2) \end{pmatrix} \quad (2)$$

Making use of Eqs. (1) and (2), Eq. (3) is derived.

$$a \begin{pmatrix} \cos \theta_A \\ \sin \theta_A \end{pmatrix} + b_1 \begin{pmatrix} -\sin(\theta_A + \phi_2) \\ \cos(\theta_A + \phi_2) \end{pmatrix} + (R+r) \begin{pmatrix} \cos(\theta_A + \phi_2) \\ \sin(\theta_A + \phi_2) \end{pmatrix} + \begin{pmatrix} 0 \\ \delta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (3)$$

The unknowns of Eq. (3) are b_1 and ϕ_2 . Eliminating b_1 from Eq. (3), Eq. (4) is obtained.

$$a \cos \phi_2 + R + r + \delta \sin(\theta_A + \phi_2) = 0 \quad (4)$$

From Eq. (4), $\cos \phi_2$ can be solved as,

$$\cos \phi_2 = \frac{-\beta_A \pm \sqrt{\beta_A^2 - \alpha_A \gamma_A}}{\alpha_A} \quad \text{where,} \quad \begin{cases} \alpha_A = a^2 + 2a\delta \sin \theta_A + \delta^2 \\ \beta_A = (R+r)(a + \delta \sin \theta_A) \\ \gamma_A = (R+r)^2 - \delta^2 \cos^2 \theta_A \end{cases} \quad (5)$$

Once the unknown ϕ_2 was derived by Eq. (5), the other unknown b_1 can be derived by substituting ϕ_2 into Eq. (3).

$$b_1 = \frac{a \cos \theta_A + (R + r) \cos(\theta_A + \phi_2)}{\sin(\theta_A + \phi_2)} \quad (6)$$

Making use of Eq. (1), the coordinates of points P_2 and P_1 are given by,

$$\left. \begin{aligned} \mathbf{P}_2 &= \begin{pmatrix} 0 \\ \delta \end{pmatrix} + a \begin{pmatrix} \cos \theta_A \\ \sin \theta_A \end{pmatrix} + r \begin{pmatrix} \cos(\theta_A + \phi_2) \\ \sin(\theta_A + \phi_2) \end{pmatrix} \\ \mathbf{P}_1 &= \mathbf{P}_2 + b_1 \begin{pmatrix} -\sin(\theta_A + \phi_2) \\ \cos(\theta_A + \phi_2) \end{pmatrix} \end{aligned} \right\} \quad (7)$$

Equations (7) are used to choose the correct answer of ϕ_2 in Eq. (5). The length of the vector $\mathbf{P}_1$ calculated from the correct ϕ_2 is R as can be seen in Fig. 3.

In a same manner, considering a closed path that consists of points O , O_P , O_B , P_7 , P_8 and O , the vector equation corresponding to Eq. (3) is

$$\begin{pmatrix} 0 \\ \delta \end{pmatrix} + a \begin{pmatrix} \cos \theta_B \\ \sin \theta_B \end{pmatrix} + (r - R) \begin{pmatrix} \cos(\theta_B - \phi_7) \\ \sin(\theta_B - \phi_7) \end{pmatrix} + b_4 \begin{pmatrix} \sin(\theta_B - \phi_7) \\ -\cos(\theta_B - \phi_7) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (8)$$

Eliminating b_4 from Eq. (8), Eq. (9) is obtained.

$$\delta \sin(\theta_B - \phi_7) + a \cos \phi_7 + r - R = 0 \quad (9)$$

From Eq. (9), $\cos \phi_7$ can be solved as,

$$\cos \phi_7 = \frac{-\beta_B \pm \sqrt{\beta_B^2 - \alpha_B \gamma_B}}{\alpha_B} \quad \text{where,} \quad \begin{cases} \alpha_B = a^2 + 2a\delta \sin \theta_B + \delta^2 \\ \beta_B = (r - R)(a + \delta \sin \theta_B) \\ \gamma_B = (r - R)^2 - \delta^2 \cos^2 \theta_B \end{cases} \quad (10)$$

Once the unknown ϕ_7 was obtained by Eq. (10), the other unknown, belt length b_4 , can be calculated by Eq. (11).

$$b_4 = -\frac{a \cos \theta_B + (r - R) \cos(\theta_B - \phi_7)}{\sin(\theta_B - \phi_7)} \quad (11)$$

Making use of Eqs. (10) and (11), the coordinates of the contact boundaries P_7 and P_8 are given by,

$$\left. \begin{aligned} \mathbf{P}_7 &= \begin{pmatrix} 0 \\ \delta \end{pmatrix} + a \begin{pmatrix} \cos \theta_B \\ \sin \theta_B \end{pmatrix} + r \begin{pmatrix} \cos(\theta_B - \phi_7) \\ \sin(\theta_B - \phi_7) \end{pmatrix} \\ \mathbf{P}_8 &= \mathbf{P}_7 + b_4 \begin{pmatrix} \sin(\theta_B - \phi_7) \\ -\cos(\theta_B - \phi_7) \end{pmatrix} \end{aligned} \right\} \quad (12)$$

Equations (12) are used to choose the correct answer of ϕ_7 in Eq. (10). The length of the vector $\mathbf{P}_8$ calculated from the correct ϕ_7 becomes R .

It is important to note that the angles ϕ_2 and ϕ_7 and belt length b_1 and b_4 vary with the rotation of power disk in an off centered condition. On the other hand, the angles ϕ_3 , ϕ_4 , ϕ_5 , ϕ_6 and belt length b_2 , b_3 remain constant regardless of the rotation of the power disk.

The angular velocity of the driven shaft ω is expressed by the summation of the angular velocity of power disk ω_p and the variation of angular velocity $\Delta\omega$ then,

$$\omega = \omega_p + \Delta\omega \quad (13)$$

Let define the variables L_1 and L_2 .

$$\left. \begin{aligned} L_1 &= b_1 + r \phi_2 \\ L_2 &= b_4 + r \phi_7 \end{aligned} \right\} \quad (14)$$

Suppose that an each length varied dL_1 and dL_2 respectively in a short time dt . The total change in the belt length of the looped belt is $dL_1 + dL_2$. Because the shaft is driven by the looped belt through two points, the belt length that affects the angular velocity variation $\Delta\omega$ is half of $dL_1 + dL_2$.

$$R \Delta\omega dt = (dL_1 + dL_2) / 2 \quad (15)$$

Equation (15) can be deformed to

$$\Delta\omega = \frac{1}{R} \frac{d}{dt} \left(\frac{L_1 + L_2}{2} \right) = \frac{1}{R} \frac{dL_{mean}}{dt} \quad \text{where,} \quad L_{mean} = \frac{1}{2} (L_1 + L_2) \quad (16)$$

Differentiation can be carried out by way of the rotational angle of the power disk θ_p ,

$$\Delta\omega = \frac{1}{R} \frac{dL_{mean}}{d\theta_p} \frac{d\theta_p}{dt} = \frac{\omega_p}{R} \frac{dL_{mean}}{d\theta_p} \quad (17)$$

Equation (17) means that when the mean belt length L_{mean} is constant regardless of the rotational angle of the power disk θ_p , the fluctuation of angular velocity $\Delta\omega$ becomes zero. Substituting Eq. (17) into Eq. (13) gives

$$\frac{\omega}{\omega_p} = 1 + \frac{1}{R} \frac{dL_{mean}}{d\theta_p} \quad (18)$$

5. Numerical Results

In order to investigate the behavior of the looped belt coupling, some calculations were made. The calculated condition is shown in each figure and calculations were carried out for every 1 degree of the power disk rotation. Figure 4 shows the change of L_1 , L_2 and L_{mean} against revolution angle of the cam-follower A in the case of $\theta_B - \theta_A = 180^\circ$. The length of L_1 and L_2 change almost in opposite phase, so the amplitude of L_{mean} is largely decreased.

The ratio of angular velocity ω/ω_p corresponding to this data can be calculated by Eq. (18). Differentiation of L_{mean} with respect to the power disk rotation angle θ_p was carried out by using 5 point formula (Wylie., 1994). Figure 5 shows the results for four kinds of off-centered distance $\delta = 0, 1, 2$ and 3mm. The amplitude of fluctuation in angular velocity increases with each increment of off-centered distance δ .

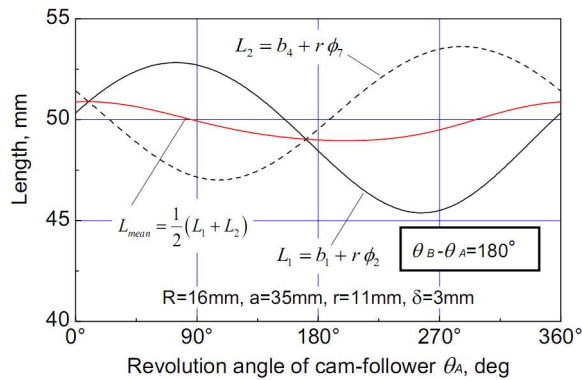


Fig. 4 Change of belt length L_1 , L_2 and L_{mean} with revolution angle of the cam-follower θ_A

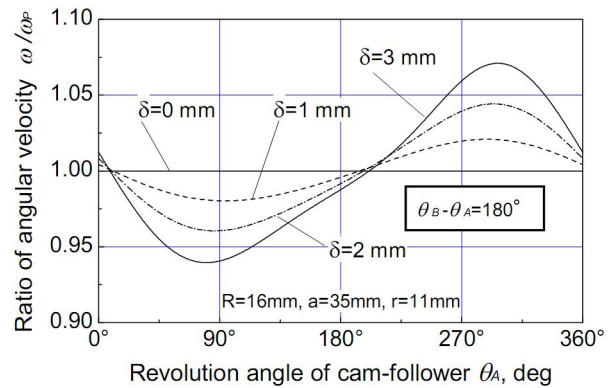


Fig. 5 Change of angular velocity ratio ω/ω_p with revolution angle θ_A for various off-centered distance δ

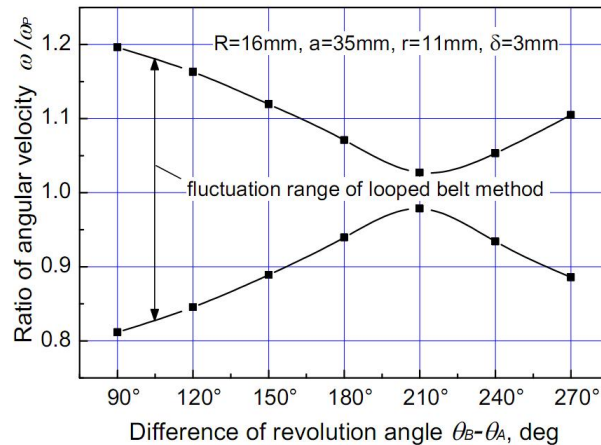


Fig. 6 Difference of revolution angle of cam-followers $\theta_B - \theta_A$ affects greatly on the fluctuation of angular velocity ratio.

It should be noted that the two opposing belts are asymmetrical even in the case where the cam-followers A and B are axial symmetry. Asymmetry installation of the looped belt may cause a fluctuation in angular velocity, so an effect of angular difference between the cam-followers A and B, $\theta_B - \theta_A$, on the fluctuation in angular velocity was investigated. There are seven kinds of angular differences from $\theta_B - \theta_A = 90^\circ, 120^\circ, \dots, 270^\circ$. Figure 6 shows the results. The upper curve consists of the maximum angular velocity ratio and the lower curve consists of the minimum angular velocity ratio in each angular difference $\theta_B - \theta_A$. It is obvious that amplitude of fluctuation is strongly influenced by the angular difference of the cam-followers $\theta_B - \theta_A$. The optimum angular difference $\theta_B - \theta_A$ exists to bring the minimum fluctuation in angular velocity under the given condition. It was $\theta_B - \theta_A = 210^\circ$ as shown in Fig. 6.

To confirm the configurations of the curve corresponding to Figs. 4 and 5 at the optimum angular difference $\theta_B - \theta_A = 210^\circ$, the change of L_1 , L_2 and L_{mean} with the revolution angle of cam-follower A, θ_A , were plotted in Fig. 7. According to Fig. 7, it can be clearly seen that shapes of L_1 and L_2 are far more axisymmetric with respect to the x axis in compared with the case of $\theta_B - \theta_A = 180^\circ$ in Fig. 4. So, their mean length L_{mean} varies to a small degree against the revolution angle of cam-follower θ_A , i.e. the rotation of power disk. The ratios of angular velocity ω/ω_P corresponding to the data in Fig. 7 were calculated by using Eq. (18). Results are shown in Fig. 8. It is noted that two kinds of coupling are included in Fig. 8. One is of the looped belt type annotated as calculated from L_{mean} . The others are of the conventional single belt type annotated as calculated from L_1 and L_2 . In the calculation for the single belt type, L_{mean} in Eq. (18) was replaced by either L_1 or L_2 . It is evident that the looped belt method was quite effective in reducing the fluctuation in angular velocity. In the case of a single belt method, the angular velocity ratio ranges from 0.772 to 1.237 for belt of L_1 , 0.792 to 1.206 for belt of L_2 . On the other hand, in the case of the looped belt method, it ranges from 0.978 to 1.027. Therefore, the magnitude of fluctuation was reduced to about only 10% compared with that of the single belt method at the optimized angular difference condition of the cam-followers, i.e. $\theta_B - \theta_A = 210^\circ$.

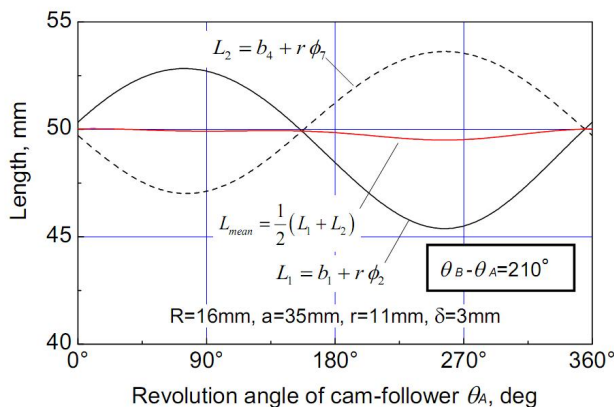


Fig. 7 Change of belt length L_1 , L_2 and L_{mean} with revolution angle of the cam-follower θ_A

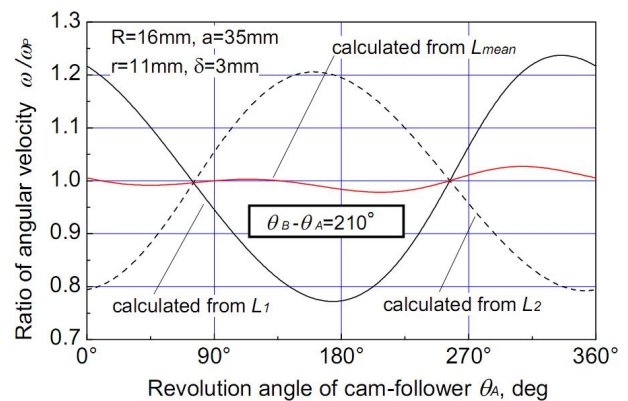


Fig. 8 Change of angular velocity ratio ω/ω_P with revolution angle θ_A calculated from L_1 , L_2 and L_{mean}

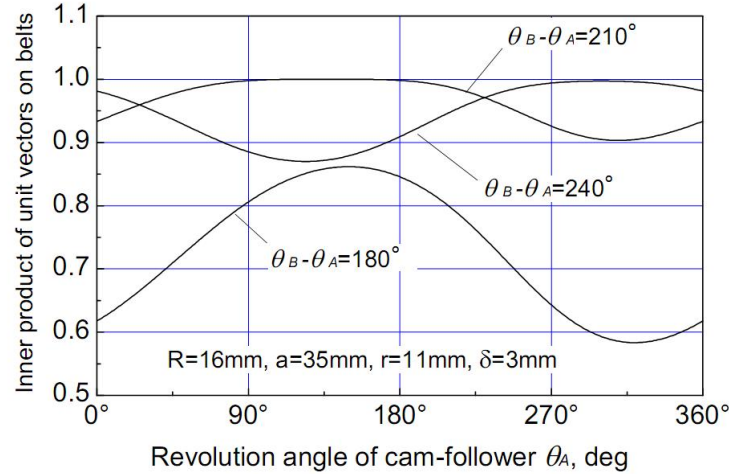


Fig. 9 Change of inner product of unit vectors on opposing belts with revolution angle of cam-follower θ_A

In order to consider the effect of direction change of the looped belt on the ratio of angular velocity ω/ω_p , inner product of unit vectors on the opposing belts defined by Eq. (19) were calculated for three kinds of difference angular conditions.

$$\frac{\overrightarrow{P_2P_1} \cdot \overrightarrow{P_8P_7}}{|\overrightarrow{P_2P_1}| |\overrightarrow{P_8P_7}|} \quad (19)$$

Results are shown in Fig. 9. When two opposing belts are in parallel, Eq. (19) becomes 1. It becomes smaller when the angle of the intersection between two unit vectors on the belts becomes large. According to the Fig. 9, it is clearly recognized that the opposing belts were most in parallel among them when the angular difference of the cam-followers was at the optimized condition, i.e. $\theta_B - \theta_A = 210^\circ$. To minimize the variation in angular velocity in an off-centered condition in the looped belt method, the direction of the two opposing belts should be parallel as much as possible to get a constant mean belt length during a rotation of the power disk. As far as this looped belt method is concerned, it might be very difficult to make a coupling that can transmit rotational torque without any variation in angular velocity under any off-centered condition. However, the change in angular velocity is small enough for practical use, so it can be used for some applications as it does not require a precise centering operation.

6. Experimental method

To confirm the theory, some experiments were carried out. Figure 10 (a) shows a test rig to measure angular velocity. Rotation was applied by a geared motor. Shaft speed was 60 r/min. Brake torque $T_b = 1.8$ Nm was applied by

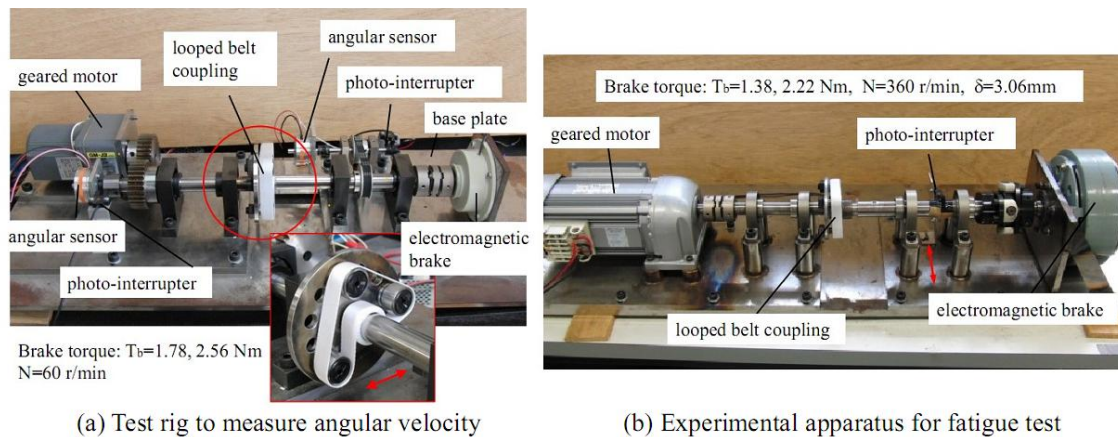


Fig. 10 Experimental apparatus of the looped belt coupling to measure angular velocity and for fatigue test

Table 1 Test condition

Item	Test rig (a)	Test rig (b)
Power, W	25	200
Torque, Nm	3.9	5.2
Shaft speed N, r/min	60	360
Brake torque T_b , Nm	1.8	1.38, 2.22
Off-centered distance δ , mm	0 ~ 3.5	3.06

both tests. The thickness of the belt is 1.2mm and 16 mm wide. The belt was sewed to make a loop by means of an industrial sewing machine. An off-centered distance was applied by sliding a base plate as shown in Figs. 10. The off-centered distance was measured by using a digital micrometer. Experimental conditions are summarized in Table 1.

7. Experimental results

The changes of angular velocity ratio with time for three kinds of angular difference of the cam-follower are shown in Figs. 11 (a), (b) and (c). Figure 11 (a) is of $\theta_B - \theta_A = 150^\circ$. Figure 11 (b) is of $\theta_B - \theta_A = 180^\circ$. Figure 11 (c) is of $\theta_B - \theta_A = 210^\circ$. Experimental curves are shown together with the corresponding theoretical ones. Some periodic characteristics can be observed in the experimental curves in Figs. 11 (a) and (b). On the other hand, the corresponding theoretical curves do not have such tendency. Because the accuracy of experimental value is not adequate, it is hard to distinguish the similarity between the experimental curve and theoretical one in Fig. 11 (c). However, from the macroscopic point of view, it seems that the experimental curves agree well with the theoretical curves. The magnitude

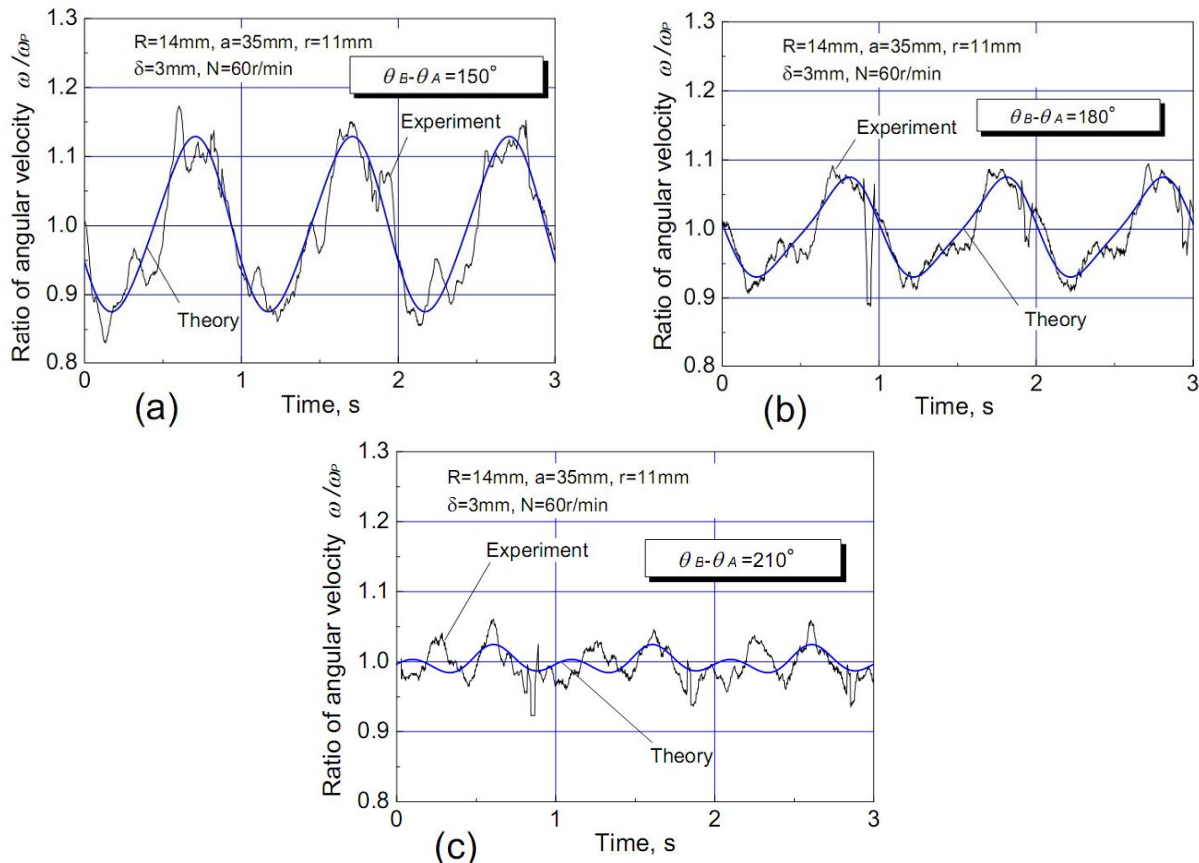


Fig. 11 Comparison of angular velocity ratio ω/ω_p between experiment and theory

an electromagnetic brake. Rotational angle of each shaft was measured by the angular position sensor (MIDORI Orange Pot α , CP-45H) with sampling rate 2ms. Angular velocity was calculated by an areal velocity method (Imado, 2013b) which gave a mean angular velocity in rotating angle within about 10° .

Figure 10 (b) shows an experimental apparatus for endurance test. Shaft speed was 360 r/min. A synthetic fiber belt was used for

of fluctuation in the ratio of angular velocity ω/ω_P becomes smaller as the angular difference, $\theta_B-\theta_A$, approaches the optimum value, i.e. $\theta_B-\theta_A=210^\circ$, as predicted by the theory. This experimental result corresponds to the theoretical results shown in Fig. 6.

Fatigue tests were also performed under the two kinds of braking torque $T_b=1.38\text{Nm}$ and 2.22 Nm with an off-centered distance $\delta=3.06\text{mm}$ by the test device shown in Fig. 10 (b). However, it was confirmed that no problem occurred up to more than 10 million cycles. Therefore, the fatigue tests were discontinued.

8. Conclusions

The looped belt method for a coupling was devised to reduce a fluctuation in angular velocity under an off-centered condition. The validity of the looped belt method was evaluated numerically and experimentally as well. Through these investigations, the following results were obtained.

- (1) The angle between the two opposing belts changes continuously during a rotation of power disk in an off-centered condition.
- (2) The angle between the two opposing belts determined by the location of the two cam-followers greatly affects the fluctuation in angular velocity in an off-centered condition for the looped belt coupling.
- (3) Change of angular velocity is determined by the change in the mean length of the two opposing belts in the looped belt coupling.
- (4) The looped belt method is very effective in reducing a fluctuation in angular velocity. The magnitude of the fluctuation in angular velocity was decreased down to about 10% at the optimum condition in compared with the former single belt coupling.
- (5) The fatigue life of the looped belt coupling was evaluated under the braking torque $T_b=2.22\text{Nm}$ and no problem occurred over the span of 10 million cycles.

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