

Investigation of dynamic behavior of planet gears in planetary gear sets for three-axis driving

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Abstract

Planetary gear set is widely used in hybrid vehicle as a power distribution system or in electric vehicle as a high reduction system. Although it has many advantages, its noise and vibration problem has emerged. There have been many reports dealing with two-axis driving or displacement of planet gear under its condition, but we can see no report dealing with about three-axis driving. As first novel point in the present report, three-axis driving is tested on the original test stand. As second novel point in this report, characteristic of instantaneous center of the planet gear is investigated based on mechanism. Planet gear rotates and revolves at the same time during working in three-axis driving condition. Instantaneous center was defined according to velocity distribution of planet gear or superposition of velocity distribution of rotation and that of revolution with simple lumped model. Tangential speed is equal to zero at the instantaneous center, and planet gear rotates around it instantaneously. Instantaneous center moves according to the driving condition or input ratio between two inputs which input from ring gear and carrier. It is very important to consider generated inertia force due to rotation around instantaneous center, to design planetary gear set for three-axis driving. The meshing force between planet gear and ring gear and the meshing force between planet gear and sun gear have same trend and close value but they are not same value, and absolute values of ring-planet meshing force are larger than those of sun-planet meshing force. It is also confirmed that the torques around planet gear are balanced in steady condition, considering both transmission and rotation.

Key words : Instantaneous center, Gear, Hybrid vehicle, Planetary gear set, Torque measurement

1. Introduction

Although planetary gear sets are widely used in various machines, its noise and vibration problem has emerged, and its dynamic characteristics are not fully understood due to the complexity. Recently, planetary gear sets is used in advanced way which dealing with two inputs to combine them into single output. Many reports have cited the problem of planet gear displacement occurring in two-axis driving. Parker et al. noted the structured vibration modes of general compound planetary gear systems (Kiracofe and Parker, 2007), used analytical and finite element models to ascertain the nonlinear dynamics of planetary gears (Ambarisha and Parker, 2007), and conducted planetary gear modal vibration experiments to identify the correlation against lumped-parameter and finite element models (Ericson and Parker, 2013). Velex reported on the dynamic simulation of eccentricity errors in planetary gears (Gu and Velex, 2013) and on a dynamic model for studying the effects of planet position errors in planetary gears (Gu and Velex, 2012). Torque sharing of planet gears based on FEM (Singh, 2010) has also been reported. However, only a few reports have dealt with dynamic characteristic in three-axis driving especially based on mechanism. There have been few reports dealing with kinematics of planet gear trains from a view of its instantaneous center under two-axis driving (Pennock and Alwerdt, 2007). There is no report dealing with centrode of planet gear trains under three driving shaft conditions. We therefore focus on the variation of centrode radius of planet gear trains as a novel parameter to estimate the transmission under three-axis

driving. In this paper, motion and balance of average forces of planet gear under steady rotation is discussed based on mechanism, or rotation and translation is considered based on its velocity distribution during three-axis rotation. The reproducibility of torque is confirmed from the experimental result and that ensures the reliability of data. The meshing force between planet gear and ring gear and the meshing force between planet gear and sun gear have same trend and close value but they are not same value, and absolute values of ring-planet meshing force are larger than those of sun-planet meshing force. That means planet gear is twisted in three-axis driving and it should be considered when planetary gear set is designed. It is also confirmed that the torques around planet gear are balanced in steady condition, considering both transmission and rotation at the same time.

2. Planetary gear set

A planetary gear set is shown in Fig.1. It has four basic components: ring gear, sun gear, planet gears, and a carrier that, connects the planet gears. Suffixes s, c, r, p correspond to the sun gear, carrier, ring gear, and planet gears. The rotational speed of each respective shaft is defined N_s , N_c , N_r and N_p . The input can be given through three components excluding the planet gears. A planetary gear set can provide a multi gear ratio by changing the input and output axis and deal with multi-inputs or multi-outputs at the same time. However, planetary gear sets have a complicated structure and consequently their dynamic characteristics are not fully understood.

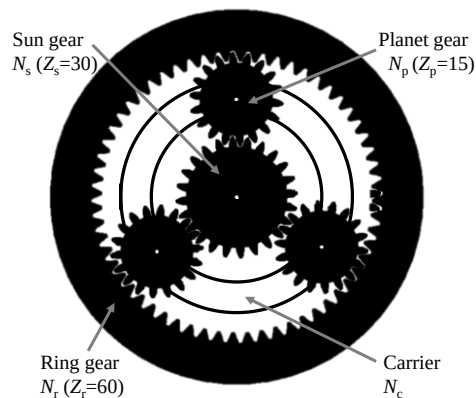


Fig.1 Planetary gear set has four basic components: ring gear, sun gear, planet gears, and a carrier that, connects the planet gears. The input can be given through three components excluding the planet gears because planet gears are hidden inside of the system and cannot be accessed.

3. Usage of planetary gear set

3.1 Automatic transmission

Planetary gear sets are widely used in stepped automatic transmissions with their advantages like multi-gear ratio, compactness, and in and out on the same axis. A single planetary gear set ordinarily provides a two-gear ratio in a stepped automatic transmission: Another ratio is used for reverse rotation. Three planetary gear sets are used in 5-speed and 6-speed transmissions. In every transmission type, planetary gear sets are used for two-axis driving. Recently, the number of automatic transmission speeds has been increasing from an efficiency aspect, in order to use the high efficiency area of internal -combustion engines, or from a friction aspect to enable down speeding. ZF has released the 9-speed transmission 9HP, which is used in Range rover (Gaertner and Ebenhoch, 2013).

3.2 Series-parallel hybrid system

Toyota uses planetary gear sets as the core of series-parallel hybrid system (Liu, et al., 2005) to combine engine power and motor power to achieve higher efficiency. In such system, planetary gear sets are used in a very complex way, not only for two-axis driving to transmit engine power to the car as with an ordinary automatic transmission, or to transmit braking energy to the generator to make electricity (called kinetic energy recovery brake), but also for three-axis driving functions like transmitting combined power from the engine and motor (called power mode), or distributing engine power to the car and generator (called charge mode). A 5-planet planetary gear set is widely used in such system. Control of the planetary gear set is the key for achieving higher efficiency. At the same time, however, the noise generated by planetary gear set has emerged as a problem due to the quietness of hybrid vehicles.

4. Planetary gear set theory

4.1 Relation of component speeds

The speed of each component is determined by synthesis of two kinds of rotation modes.

$$N_s : N_p : N_r : N_c = 1 : -\frac{Z_s}{Z_p} : -\frac{Z_s}{Z_r} : 0 \quad (1)$$

$$N_s : N_p : N_r : N_c = 1 : 0 : 1 : 1 \quad (2)$$

In these Equations, N_r is defined as a positive value and a negative value means opposite rotation direction. Eq. (1) is for star mode and Eq. (2) for rigid rotation mode. The relation of rotation speed can be calculated by solving these Equations with method of indeterminate coefficients.

$$\begin{cases} N_p = \frac{Z_s Z_r}{Z_p (Z_s + Z_r)} (N_r - N_s) = \frac{Z_r}{Z_p} (N_r - N_c) = \frac{Z_s}{Z_p} (N_c - N_s) \\ N_c = \frac{1}{Z_s + Z_r} (Z_s N_s + Z_r N_r) \end{cases} \quad (3)$$

As shown in Eq. (3), the rotation speed of planetary gear N_p is determined by the difference in rotation speeds between any two other components and the gear ratio of the planetary gear set. Then, introduce γ as the ration between N_r and N_c , or $\gamma = N_c/N_r$,

$$N_p = \frac{Z_r}{Z_p} N_r (1 - \gamma) \quad (4)$$

$$N_s = \frac{N_r}{Z_s} \{ (Z_s + Z_r) \gamma - Z_r \} \quad (5)$$

From Eq. (4) and (5), the rotation condition can be described like in Table 1.

Table 1 Driving condition of planetary gear set

γ	a<	0	<b<	$\frac{Z_r}{Z_r + Z_s}$	<c<	1	<d
N_c	-	0	+	+	+	+	+
N_p	+	+	+	+	+	0	-
N_s	-	-	-	0	+	+	+

In Table 1, the ring gear rotation direction is defined as a positive value, i.e., described as +. According to this table, the driving condition of planetary gear set can be divided into 4 special conditions and another 4 general conditions. As the special driving conditions, when the $\gamma=0$, planetary gear set is in the star mode (carrier fixed), when the $\gamma=Z_r/(Z_r+Z_s)$, it is in the solar mode (sun gear fixed), and when the $\gamma=\infty$, it is in the planet mode (ring gear fixed). These three conditions are called two- axis driving and these are special condition of three-axis driving. And there is another special condition. When the $\gamma=1$, planetary gear set is in the rigid rotation mode. In that condition all components except planet gear rotate in the same direction at the same speed as a lump. And rest ones from a to d are 4 general conditions of three-axis driving.

4.2 Meshing force on tooth surface

The geometry of an involute gear results in its having a pitch circle and a base circle. The meshing force can be described as shown in Fig.2.

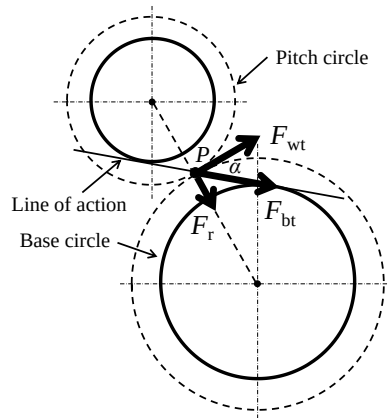


Fig.2 The ISO defines meshing force as the meshing force on the common tangential line of the pitch circle. It is called nominal tangential force F_{wt} and is vertical to the axis of the gear. On the other hand, meshing occurs on the common tangential line of the base circle (called the line of action). The meshing force on that line is defined as F_{bt} , which corresponds to the actual meshing force and is vertical to the tooth surface.

The relation between F_{wt} and F_{bt} is as in Eq. (6).

$$F_{wt} = F_{bt} \cos \alpha \quad (6)$$

Here, α is the normal (20°) pressure angle. There is radial force F_r as shown in Fig.2, but it means nothing in terms of engineering and F_{wt} corresponds to the axial force.

The radii of the pitch circle and base circle are respectively defined as in Eq. (7) and (8).

$$R = \frac{mz}{2} \quad (7)$$

$$R_b = \frac{mz}{2} \cos \alpha \quad (8)$$

F_{wt} is calculated as follows by torque and pitch radius.

$$F_{wt} = \frac{T}{R} \quad (9)$$

Two kinds of planetary gear set models are conceivable, i.e., a model with a pitch circle and one with a base circle. In this paper we refer to the model of the former as a simple lumped parameter model. Nominal meshing force can be taken into consideration in this model; it is sufficient to consider the balance around the planet gears. In the model, F is nominal meshing force as F_{wt} , i.e., the force tangential to the pitch circle. The respective forces from the ring gear, sun gear and carrier to the planet gears are F_{pr} , F_{ps} , and F_{pc} , respectively.

Here, it is well known that $F_{pr} = F_{ps}$ in two-axis driving conditions, i.e., star mode (carrier fixed), solar mode (sun gear fixed) and planet mode (ring gear fixed) based on simple modeled calculation as below. Then relation between F_{pc} and F_{pr} , F_{ps} is $F_{pc} = F_{pr} + F_{ps} = 2 F_{pr}$ or $2 F_{ps}$.

However, $F_{pr} \neq F_{ps}$ under three-axis driving condition to transfer power to each axis. Each meshing force F_{pc} , F_{pr} and F_{ps} is uniquely determined under the relation $F_{pc} + F_{pr} + F_{ps} = 0$. This is biggest difference between three-axis driving and two-axis driving.

According to Table 1, meshing forces around the planet gear for 4 general driving conditions are considered as in Fig.3. In the model, original point O on the XY surface is the rotation center of the planetary gear set. O_p is the actual

rotation center of planet gear, X_p axis and Y_p axis are tangential and radius direction respectively.

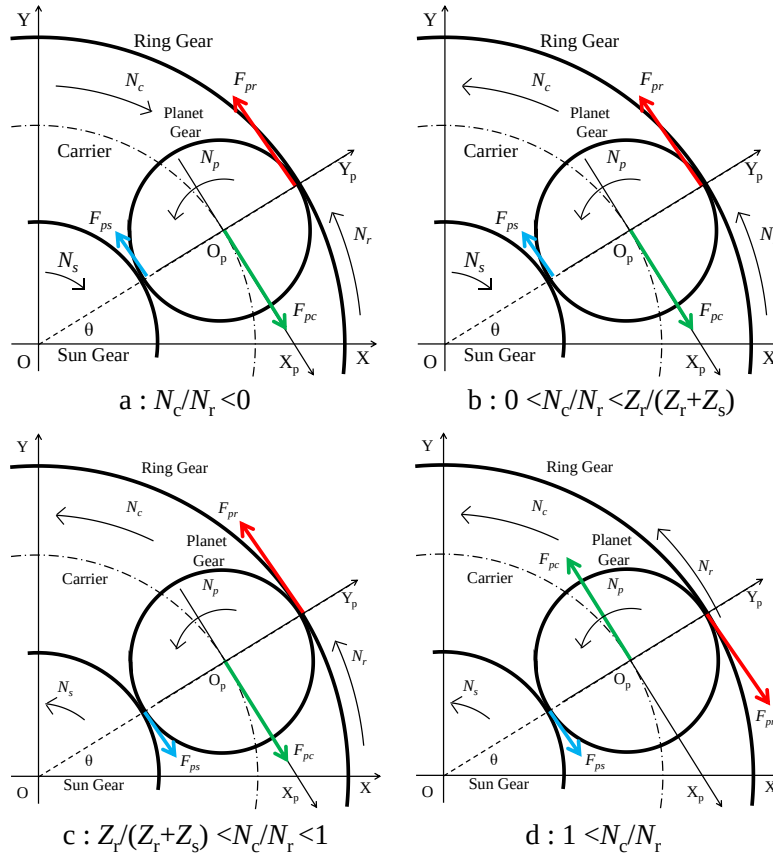


Fig.3 The relations of meshing forces differ depend on driving conditions as shown. F is nominal meshing force, i.e., the force tangential to the pitch circle. The former suffix is the element that is given the meshing force and the latter suffix is the one that gives the meshing force. The respective forces from the ring gear, sun gear and carrier to the planet gears are F_{pr} , F_{ps} , and F_{pc} , respectively.

4.3 Instantaneous rotation center

It is important to consider both rotation and translation, or motion to sideways, in the motion of rigid body by mechanism. In the three-axis driving, the planet gears not only rotate but also revolve at the same time. The velocity distribution of the planet gears' rotation is shown in Fig.4. In this paper, we define tangential speed as a positive value for the CCW (Counter Clockwise) of planet gear. In the figure, V_{sp} is the velocity at the sun gear-side and V_{rp} is that at the ring gear-side of the planet gears turning at the speed of N_p . The speeds can be described as in Eq. (10).

$$V_{rp} = V_{sp} = R_p \omega_p = \frac{mZ_p}{2} \cdot \frac{2\pi}{60} \frac{Z_r}{Z_p} N_r (1 - \gamma) = \frac{m\pi}{60} Z_r N_r (1 - \gamma) \quad (10)$$

This Equation shows that the velocity at the two sides is the same but the direction is opposite. Furthermore, the revolution speed varies according to the distance from the rotation center O , as shown in Fig.5. The revolution of the planet gears is Equal to the rotation of the carrier. In the figure, V_{sc} is the velocity at the sun gear-side and V_{rc} is that at the ring gear-side of the carrier turning at the speed of N_c . These speeds are described as following Eq. (11) and (12).

$$V_{rc} = R_r \omega_c = \frac{m\pi}{60} Z_r N_r \gamma \quad (11)$$

$$V_{sc} = R_s \omega_c = \frac{m\pi}{60} Z_s N_r \gamma \quad (12)$$

Eq. (11) and (12) show that V_{rc} is faster than V_{sc} because of the difference in the pitch radius, or the difference of the number of teeth.

When a planet gear rotates and revolves at the same time, the velocity distribution is like that shown in Fig.6, or the combination of that shown in Figs.4 and 5.

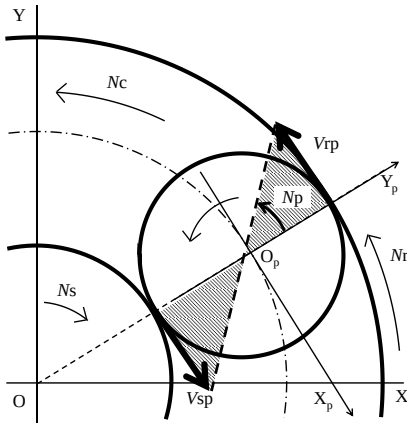


Fig.4 Velocity distribution of rotation

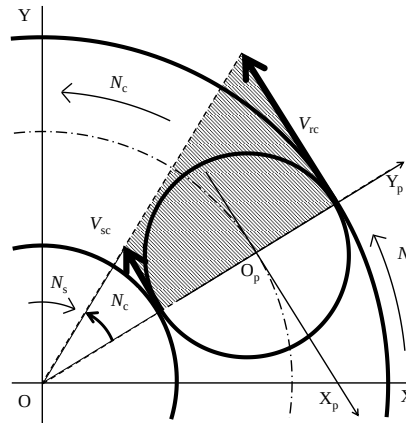


Fig.5 Velocity distribution of revolution

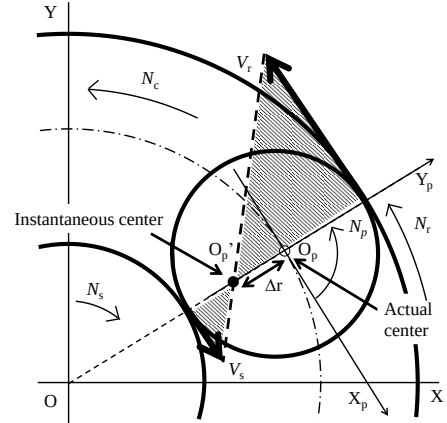


Fig.6 Total velocity distribution

The total meshing speed V_s and V_r , respectively the sun gear-side and ring gear-side velocity of the planet gear, is given like in Eq. (13).

$$\begin{cases} V_r = V_{rp} + V_{rc} = \frac{m\pi}{60} Z_r N_r \\ V_s = V_{sp} - V_{sc} = \frac{m\pi}{60} N_r \{Z_r - (Z_r + Z_s)\gamma\} \end{cases} \quad (13)$$

Instantaneous rotation center O_p' is the point at which its velocity is zero on the Y_p axis. The planet gear rotates around O_p' instantaneously. From the ratio between V_s and V_r , the difference Δr between the rotation center O and instantaneous rotation center O_p' is defined as follows.

$$\Delta r = -R_p \frac{(Z_r + Z_s)\gamma}{2Z_r - (Z_r + Z_s)\gamma} \quad (14)$$

O_p is the original point of Δr and the positive direction of Δr accords with Y_p axis'. The ratio ε , i.e., the distance from the rotation center O and instantaneous rotation center O_p' and that between the actual rotation center of the planet gear is calculated in Eq. (15).

$$\varepsilon = \frac{(R_s + R_p) + \Delta r}{R_s + R_p} = 1 + \frac{\Delta r}{R_s + R_p} = 1 - \frac{Z_p}{Z_s + Z_p} \cdot \frac{(Z_r + Z_s)\gamma}{2Z_r - (Z_r + Z_s)\gamma} \quad (15)$$

Here, R_p and R_s are the pitch radius of the planet gear and sun gear, respectively. The ε values calculated with the input speed ratio γ are shown in Fig.7.

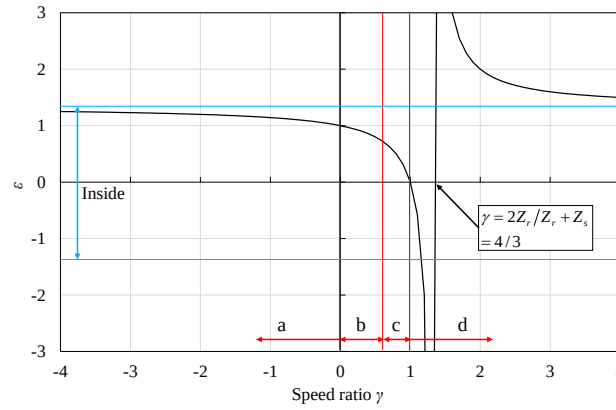


Fig.7 Instantaneous center moves as shown according to speed ratio between ring gear and carrier. It exists in the inside of the system in driving condition a, b and c. On the other hand, there is an asymptote in driving condition d and instantaneous center rarely exists in the inside of the system.

There is an asymptote in the figure and when the speed ratio is close to it, ε goes to infinity as shown in Eq. (16).

$$\left(\begin{array}{l} \lim_{\gamma \rightarrow \frac{2Z_r}{Z_r + Z_s} + 0} \varepsilon = +\infty \\ \lim_{\gamma \rightarrow \frac{2Z_r}{Z_r + Z_s} - 0} \varepsilon = -\infty \end{array} \right. \quad (16)$$

When the speed ratio goes to infinity, ε approaches to 1 as shown in Eq. (17).

$$\lim_{\gamma \rightarrow \pm\infty} \varepsilon = \frac{Z_r}{Z_s + Z_p} \quad (17)$$

When a planetary gear set is in star mode or its speed ratio is equal to zero, radius ratio ε is equal to 1. This means the instantaneous rotation center corresponds to the actual rotation center of the planet gear. When it is in lumped rotation mode or the speed ratio is equal to 1, radius ratio ε is equal to 0. This means the instantaneous rotation center corresponds to the rotation center of the sun and ring gears, or that of the planetary gear set itself. As shown in Eq. (16), when the speed ratio is at the asymptote, the instantaneous rotation center goes to infinity. In this case the planet gear translates, or goes sideways.

5. Experimental results and discussion

5.1 Experimental setup

The prototype equipment we used to estimate three axis driving is shown in Fig.8. The torque of each axis was measured by using (a) a UTM-3Nm (Uni pulse, Inc.) torque meter and the rotation pulse was measured by using an EE-SX870-2M (Omron) photo micro sensor. The torque meter specifications were 25,000 min^{-1} maximum speed, ± 3 Nm maximum torque, 1/10000 resolution and 200 Hz cutoff frequency. A radio type wireless accelerometer ASH-A-500 (Tateyama Denshi) called an FM telemeter was put on the carrier to record acceleration.

A 3-planet set and 6-planet set were used for this experiment. The gear geometry factors are shown in Table 2. The input was given through both the ring gear and the carrier as Table 3, and the output through sun gear N_s . The brake torque was input on the sun gear axis by Hysteresis brake.

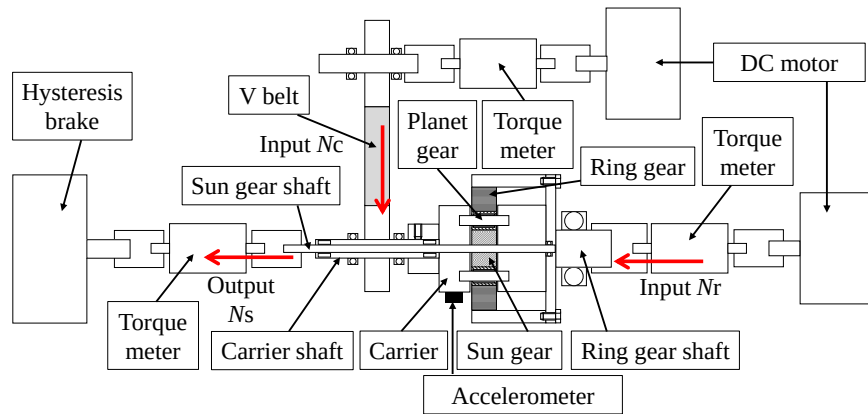


Fig.8 The input was given through both the ring gear and the carrier through DC motors (rating torque: 0.4Nm, Oriental Motor, Inc.), and output to sun gear. Brake torque was loaded on sun gear by hysteresis brake AHB-3 (Magtrol, Inc.). Torque and rotation pulse of three-axis except planet gear were measured.

Table 2 Gear geometry

	Sun	Planet	Ring	Carrier
Module [mm]	1			---
Number of teeth	30	15	60	---
Normal pressure angle [deg]	20			---
Helix angle [deg]	0			---
Tooth depth coefficient [mm]	2.25			---
Tooth width [mm]	6	10	10	---
Pitch diameter [mm]	30	15	60	---
Base diameter [mm]	28.2	14.1	56.4	---
Mass [kg]	0.06	0.027	0.272	0.752
Inertia [kgmm ²]	6.75	0.76	610.8	397.8
Material	S45C			
Heat treatment	None			

Table 3 Experimental condition

Driving condition	γ_1	γ_2	γ_3
a	-0.6	-0.4	-0.2
b	0.2	0.4	0.6
c	0.7	0.8	0.9
d	1.2	1.4	1.6

5.2 Confirmation of the reproducibility of torque

It is important to confirm the reproducibility first of all because it is thought that meshing pair of planetary gear set is restored with its cycle like simple gear train. In star mode (carrier fixed), planetary gear set is considered as simple gear train and its cycle time is determined the time ring gear rotates around carrier because the number of teeth of ring gear is always bigger than that of sun gear in planetary gear set. Then, the cycle time of three-axis driving T_{cy} can be defined as the time ring gear rotates relatively around carrier as below.

$$T_{cy} = \frac{L.C.M(Z_r, Z_s)}{Z_r} \cdot \frac{60}{|N_r - N_c|} = \frac{L.C.M(Z_r, Z_s)}{Z_r} \cdot \frac{60}{N_r |1 - \gamma|} \quad (18)$$

Calculated cycle time based on Eq. (18) is shown in Fig.9.

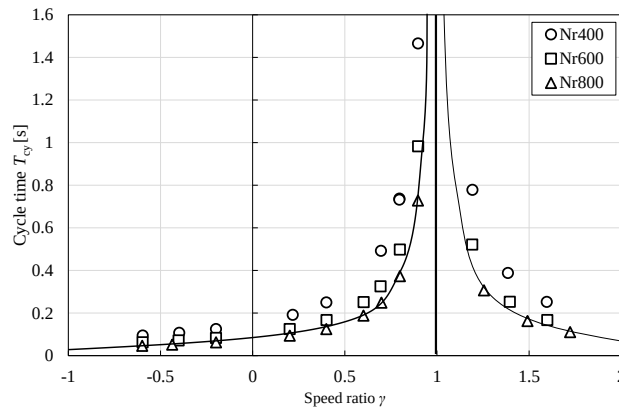


Fig.9 As shown in Eq. (18), there is asymptote when $\gamma=1$, or rigid body mode. It is natural that cycle time of rigid body mode is infinity because there is no meshing due to same meshing pair. And cycle time T_{cy} goes close to Zero in two situations when carrier rotates in opposite direction to ring gear and when carrier rotates faster than ring gear. (Input torque: 0.4Nm, Brake torque: 0.05Nm)

The fact that meshing pair of planetary gear set is restored with its cycle means the wave shape of measured torque has reproducibility for each cycle. Fig.10 shows the reproducibility of ring gear torque under $N_r=600 \text{ min}^{-1}$, $\gamma=0.4$ ($T_{cy}=0.167\text{s}$). The reproducibility of torque is confirmed from this figure, and that shows enough reliability of this data.

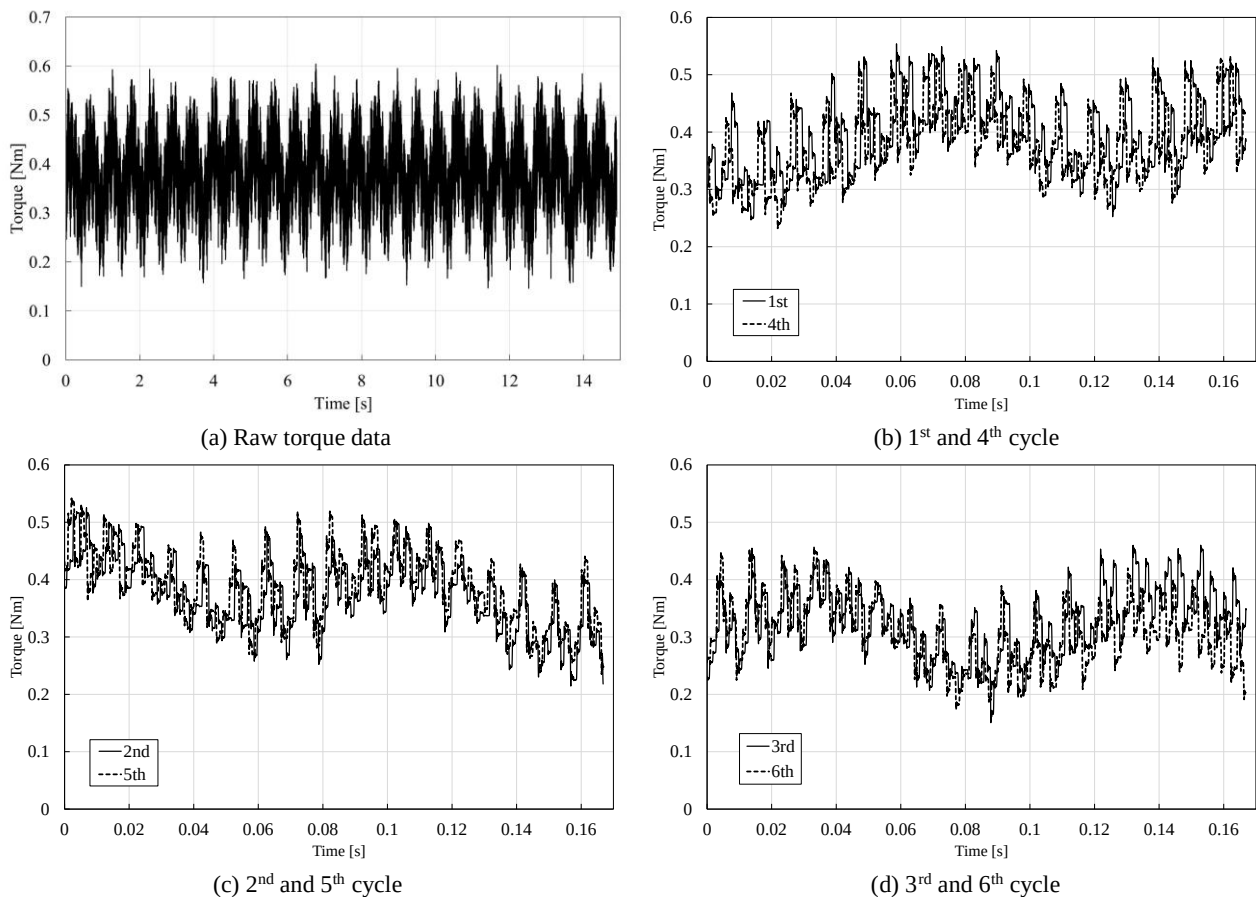


Fig.10 Reproducibility clearly exists in (a) raw data. When analyze reproducibility in each cycle in detail, it was found that there are reproducibility in every 3cycles as shown in (b), (c) and (d). The wave shapes are completely same and all small peaks are almost same in every 3cycles not every each cycle. (Input torque: 0.4Nm, Brake torque: 0.05Nm)

Then, average torque is dealt in this paper to simplify the data and to grab the phenomena what happens around planet gear in steady condition.

5.3 Balance around planet gear

Result of meshing forces in Fig.3 are shown in Fig.11, calculated in Eq. (9) with the average of measured torque. And its balances are shown in Fig.12.

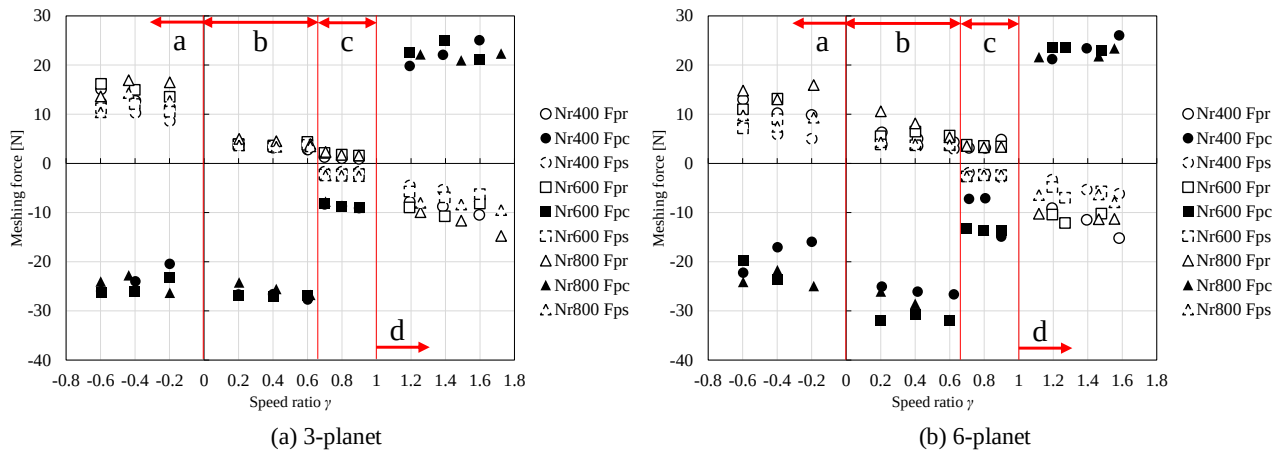


Fig.11 Result of meshing forces shows that there is no velocity dependence among the meshing forces, and there is no dependence on number of planet gears as well. In every driving condition, F_{pc} , or force from carrier to planet is the largest force among the three. Although it was generally thought that planet gear transfers internal force in previous researches (Morozumi, 1971), Fpr and Fps have same trend and close value but they are not same value. Absolute values of F_{pr} are larger than that of F_{ps} except driving condition (c).

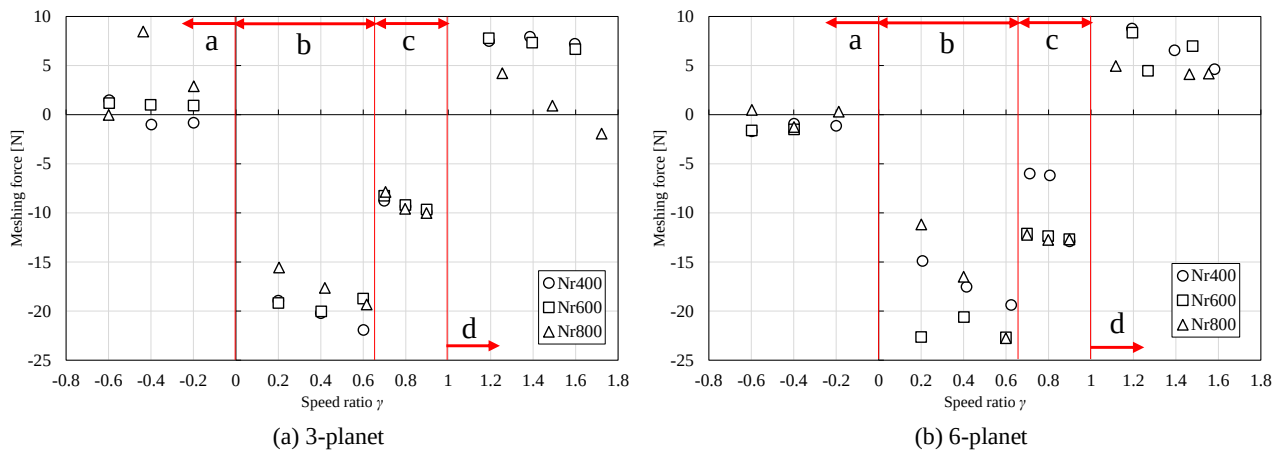


Fig.12 Balance of meshing forces shows that, in every result, X_p direction transitional forces are not balanced at all in Fig.3. Especially in driving condition (b), the balance is most negative, and in driving condition (c) as well. And balance in driving condition (d) is positive. On the other hand, it is well balanced in driving condition (a). These facts mean that the balance of forces are different in three-axis driving compared to that in two-axis driving.

Results above in Fig.11 and 12 mean that planet gear is twisted under three-axis driving. Then, balance of torque around instantaneous rotation center, considering the motion of planet gear around instantaneous center like in Fig.13.

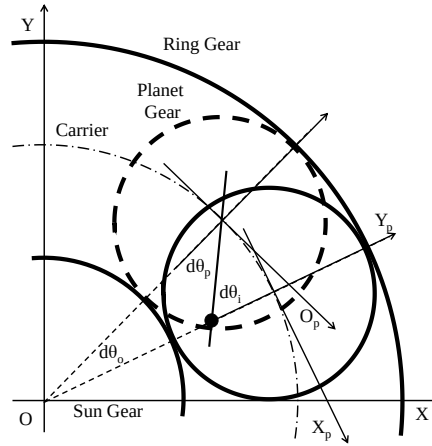


Fig.13 When planet gear rotate around instantaneous center, its motion includes both rotation and revolution. The relation of angles is quite simple like exterior angle theorem: $d\theta_i = d\theta_o + d\theta_p$. If planet gear rotate by $d\theta_i$ around instantaneous center, $d\theta_o$ is transitional part and $d\theta_p$ is rotational part.

The equations of motion are constructed in both transitional part and rotational part based on $I\ddot{\theta} = T$. The equation of transitional part is as below.

$$(I_c + n \cdot M_p \cdot R_c^2) \ddot{\theta}_o = F_{pr} \cdot R_r + F_{pc} \cdot R_c + F_{ps} \cdot R_s \quad (19)$$

The Equation of rotational part is as follows.

$$I_p \ddot{\theta}_p = F_{pr} \cdot R_p + F_{ps} \cdot R_p \quad (20)$$

Then total equation of motion is constructed as follows because $\ddot{\theta}_i = \ddot{\theta}_o + \ddot{\theta}_p$.

$$\begin{aligned} (I_p + M_p \cdot \Delta r^2) \ddot{\theta}_i &= (I_p + M_p \cdot \Delta r^2) (\ddot{\theta}_o + \ddot{\theta}_p) \\ &= (I_p + M_p \cdot \Delta r^2) \left\{ \frac{F_{pr} \cdot R_r + F_{pc} \cdot R_c + F_{ps} \cdot R_s}{I_c + n \cdot M_p \cdot R_c^2} + \frac{F_{pr} \cdot R_p + F_{ps} \cdot R_p}{I_p} \right\} \end{aligned} \quad (21)$$

Here, n is number of planet gears ($n=3$ in this paper) and M_p is mass of planet gear. The value of Eq. (21) should be zero in steady rotation. The result of Eq. (21) is shown in Fig.14.

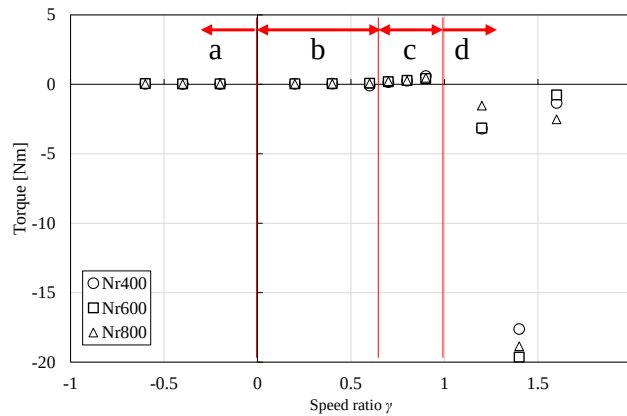


Fig.14 The result of torque balance around instantaneous rotation center shows that its value are equal to zero except driving condition (d) because according to Fig.7, instantaneous center is far from actual center and the inertia in Eq. (21) gets larger especially $\gamma=1.4$. This results mean that the meshing forces, or the torques around planet gear are balanced in steady condition, considering both transmission and rotation at the same time.

6. Conclusion

The instantaneous center is found to be an effective parameter to estimate the planet gear trains under three-axis driving. In this paper, meshing forces around planet gear are focused on and the respective forces from the ring gear, sun gear and carrier to the planet gears are F_{pr} , F_{ps} , and F_{pc} , respectively. The obtained conclusions are follows.

1. Planet gear rotates and revolves at the same time in the three-axis driving, and it rotates around instantaneous center continuously.
2. The reproducibility of torque is confirmed in every several cycles not every each cycle.
3. F_{pr} and F_{ps} have same trend and close value but they are not same value. Absolute values of F_{pr} are larger than those of F_{ps} .
4. Inertia force is generated due to rotation around instantaneous center under three-axis driving. It is very important to consider such inertia force to design planetary gear set for three-axis driving.
5. The torques around planet gear are balanced in steady condition, considering both transmission and rotation.

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