

Estimation method of support reaction on face sheet of sandwich panel by inverse analysis of deformed shape data

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Abstract

When local compressive load is vertically applied on the face sheet of the sandwich panel, the face sheet will be deflected and the core layer will be crushed at relatively low load. In order to evaluate the mechanical property of the sandwich panel subjected to the local compression, it is important to estimate the reaction on the face sheet supported by the core layer. However it is difficult to directly measure the support reaction because the core layer has been crushed. This paper describes a novel method to estimate the support reaction on the face sheet of the sandwich panel by inverse analysis of the deformed shape data of the face sheet. In this method, the finite element method based on strain increment theory is applied to the face sheet using the measured deflection of the face sheet as the boundary condition. Then it estimates the nodal load of the finite element model of the face sheet that is equivalent to the support reaction by the Tikhonov regularization. In order to demonstrate the validity of the proposed method, numerical simulations have been performed. In the numerical simulations, the several support conditions of the face sheet were given in advance. Then the direct analysis was applied to obtain the correct support reaction and the deformed shape of the face sheet in each condition. The inverse analysis was applied on the simulation data of the deformed shape to estimate the support reaction. The results show that the support reaction estimated by the proposed method has good agreement with the correct value. Moreover, it is concluded that the proposed method requires the location of the support point as a priori information.

Key words : Inverse problem, Finite element method, Sandwich panel, Laminated construction, Shell structure, Structural mechanics, Deformation characteristics, Materials and structure diagnosis

1. Introduction

Sandwich panels such as honeycomb core sandwich panel, roll core sandwich panel and foam core sandwich panel are types of structural panels where one core layer is bonded in between two face sheets. These panels have high specific strength and high specific rigidity while maintaining light weight. In addition, they are superior in impact absorption when the uniform compressive load is applied over the entire face sheet, because of the buffer effect of the core layer (Ogasawara, et al., 2010; Miller, et al., 2011). On the other hand, when local compressive load such as drop weight impact is vertically applied on the face sheet of the sandwich panel, the face sheet will be deflected and the core layer will be crushed near the point of action at relatively low load (Rathbun, et al., 2006; Kobayashi, et al., 2015). Then the face sheet is supported by the core layer (Kishimoto, et al., 2012, 2013). Therefore, in order to evaluate the mechanical property of the sandwich panel subjected to the local compression, it is important to estimate the distribution of the support reaction by the core layer.

There has been a great discussion about the approach that finite element method is applied to estimate mechanical behaviors of sandwich panels. Recent studies have suggested modeling methods of core layers for the improvement of the finite element method, as represented by the multi-scale analysis (Navarro, et al., 2013; Catapano and Montemurro, 2014; Mu, et al., 2015). Several studies tend to take the approach that the conventional finite element method is utilized for the analysis of the foam core sandwich whose core layer is isotropic and homogeneous (Styles, et al., 2008; Xu, et

al., 2014) and the analysis of the corrugated sandwich panel whose core layer is relatively simple structure (St-Pierre, et al., 2015). Although these studies have often compared calculation results with experiment data of overall mechanical behaviors such as load-displacement curve, little study has been done to actually measure the load or the stress in the sandwich panel including the distribution of support reaction on the face sheet.

However, it is difficult to directly measure the stress or the strain of the crushed core layer in actual situation. Recently, some studies have developed an evaluation method of mechanical properties between constituent members by the inverse analysis using the displacement history (Arai, et al., 2011, 2014; Kishimoto, et al., 2015a). These methods have advantages that it is unnecessary to directly equip measurement instruments on the measured object, and the deformed shape of the object which can be measured remotely is utilized for the evaluation. This study has attempted to apply this approach on the estimation of support reaction in the sandwich panel.

This paper describes a novel method to estimate the support reaction on the face sheet of the sandwich panel by inverse analysis of the deformed shape data of the face sheet. In the proposed method, the finite element method based on strain increment theory is applied to the face sheet model using the measured deflection of the face sheet as the boundary condition. Then it estimates the nodal load of the face sheet that is equivalent to the support reaction by the Tikhonov regularization. There are two main reasons why the Tikhonov regularization is applied. The first reason is that the given boundary condition is excess or lacking depending on the analysis object. The second reason is that it is necessary to reduce the influence of the measurement error in the deformed shape data on the estimated value.

In order to demonstrate the validity of the proposed method, numerical simulations have been performed. In the numerical simulations, the correct support condition of the face sheet or the correct material constants of the core layer was given in advance, and the direct analysis was applied to obtain the simulation data of the deformed shape of the face sheet. Then the inverse analysis was applied on the simulation data.

2. Estimation method of support reaction by inverse analysis

2.1. Observation equation

In the inverse analysis, the finite element method is applied on each member such as the face sheet or the core layer. Here the nodal displacement or the nodal load on the surface of the face sheet including the surface bonded with the core layer is given as the boundary condition. Then the nodal load on the surface of the face sheet bonded with the core layer is equivalent to the reaction on the face sheet supported by the core layer. According to the finite element method based on strain increment theory, the total stiffness equation for the face sheet is expressed as follows.

$$[K]\{du\} = \{df\} \quad (1)$$

where $\{du\}$ and $\{df\}$ are the displacement increment vector and the load increment vector of the nodes in the finite element model of the face sheet, respectively. The matrix $[K]$ is the total stiffness matrix.

In the displacement increment vector $\{du\}$, such as the displacement of the reference of displacement or the axis of symmetry, the vector whose components are known and forced displacements is expressed as $\{du_F\}$. The displacement increment vector excluding the vector $\{du_F\}$ is expressed as $\{du_R\}$. Similarly, in the load increment vector $\{df\}$, the vector whose components are known and forced loads is expressed as $\{df_F\}$, such as the load on the free surface. The vector whose components are estimated by the proposed method is expressed as $\{x\}$. Then Eq. (1) is expressed as follows.

$$\begin{bmatrix} [I_F] & [O] \\ [K_{RF}] & [K_R] \end{bmatrix} \begin{Bmatrix} \{du_F\} \\ \{du_R\} \end{Bmatrix} = \begin{Bmatrix} \{df_F\} \\ \{x\} \end{Bmatrix} \quad (2)$$

where the matrices $[I_F]$, $[O]$, $[K_{RF}]$ and $[K_R]$ are block matrices which correlate the vectors $\{du_F\}$, $\{du_R\}$, $\{df_F\}$ and $\{x\}$. Especially, the matrices $[I_F]$ and $[O]$ are unit matrix and zero matrix, respectively. Equation (2) is solved for the vector $\{du_R\}$ and the following equation is derived.

$$\{du_R\} = [K_R]^{-1} \begin{Bmatrix} \{df_F\} \\ \{x\} \end{Bmatrix} - [K_R]^{-1} [K_{RF}] \{du_F\} \quad (3)$$

In the left-hand side of Eq. (3), the displacement increment vector whose components are obtained by the deformed shape data is expressed as $\{du_M\}$. The vector composed of the other components is expressed as $\{du_N\}$. By using these vectors,

Eq. (3) is expressed as follows.

$$\begin{Bmatrix} \{du_N\} \\ \{du_M\} \end{Bmatrix} = \begin{bmatrix} [A_{NF}] & [A_{NX}] \\ [A_{MF}] & [A] \end{bmatrix} \begin{Bmatrix} \{df_F\} \\ \{x\} \end{Bmatrix} - \begin{bmatrix} [A_{NF}] & [A_{NX}] \\ [A_{MF}] & [A] \end{bmatrix} [K_{RF}] \{du_F\} \quad (4)$$

where

$$[K_R]^{-1} = \begin{bmatrix} [A_{NF}] & [A_{NX}] \\ [A_{MF}] & [A] \end{bmatrix} \quad (5)$$

and the matrices $[A_{NF}]$, $[A_{NX}]$, $[A_{MF}]$ and $[A]$ are block matrices which correlate the vectors $\{du_M\}$, $\{du_N\}$, $\{df_F\}$ and $\{x\}$. From Eq. (4), the observation equation that describes the direct relation between the load increment estimated by the proposed method and the displacement increment obtained by the deformed shape data is obtained as follows.

$$\{b\} = [A] \{x\} \quad (6)$$

where

$$\{b\} = \{du_M\} + \begin{bmatrix} [A_{MF}] & [A] \end{bmatrix} [K_{RF}] \{du_F\} - [A_{MF}] \{df_F\} \quad (7)$$

In the above process, it is unnecessary that the dimension of the vector $\{du_M\}$ or $\{b\}$ is equal to the dimension of the vector $\{x\}$ because the Tikhonov regularization is applied to solve the observation equation in the proposed method. Therefore the matrix $[A]$ is not necessarily square matrix.

2.2. Tikhonov regularization

If the nodes corresponding to the vector $\{du_M\}$ is the same as those of the vector $\{x\}$, all boundary conditions of Eq. (1) are activated. The solution of the vector $\{x\}$ can be obtained when the vector $\{du_M\}$ is just substituted into Eq. (1) or Eq. (2). However the size of the matrix $[A]$ is not always square matrix. In the matrix $[A]$, the fact that the number of the row is larger than the number of the column means overabundance of the boundary condition. On the other hand, the boundary condition is lacking when the number of the row is fewer than the number of the column. Moreover the condition number of the matrix $[A]$ in Eq. (6) is often large value. The disturbance of the vector $\{du_M\}$ such as the measurement error contained in the deformed shape data is enlarged and added to the solution of the vector $\{x\}$. According to references (The Japan Society of Mechanical Engineers ed., 1991; Kubo, 1992), the enlargement factor of the disturbance ratio of the vector $\{du_M\}$ for that of the vector $\{x\}$ is equal to or less than the condition number of the matrix $[A]$, expressed as follows.

$$\sqrt{\frac{\{\Delta x\}^T \{\Delta x\}}{\{x\}^T \{x\}}} \leq (\text{cond}[A]) \sqrt{\frac{\{\Delta b\}^T \{\Delta b\}}{\{b\}^T \{b\}}} \quad (8)$$

where $\{\Delta x\}$, $\{\Delta b\}$ and $\text{cond}[A]$ are the disturbance of the vector $\{x\}$, the disturbance of the vector $\{b\}$ and the condition number of the matrix $[A]$, respectively. The superscript T denotes matrix transposition.

In this study, the Tikhonov regularization is applied to reduce the influence of the above problems. The Tikhonov regularization selects the solution of the vector $\{x\}$ which minimizes the following evaluation function Π .

$$\Pi = (\{b\} - [A] \{x\})^T (\{b\} - [A] \{x\}) + \alpha \{x\}^T \{x\} \quad (9)$$

where α is the weight between the first term and the second term named Tikhonov regularization parameter. The first term means the residual sum of squares of the observation equation (Eq. (6)) and the second term means the regularization term. The vector $\{x\}$ which minimizes the evaluation function Π is derived by using the singular value decomposition of the matrix $[A]$ as follows.

$$\{x\} = [V] ([S]^T [S] + \alpha [I])^{-1} [S]^T [U]^T \{b\} \quad (10)$$

where

$$[A] = [U][S][V]^T \quad (11)$$

The matrices $[U]$ and $[V]$ are orthonormal matrices ($[U]^{-1} = [U]^T$ and $[V]^{-1} = [V]^T$), and the matrix $[I]$ is unit matrix. The matrix $[S]$ is composed of the singular values of the matrix $[A]$ and expressed as follows. When the dimension of the vector $\{b\}$ is larger than that of the vector $\{x\}$,

$$[S] = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \\ 0 & \cdots & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix}, \quad (\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n > 0) \quad (12)$$

Otherwise,

$$[S] = \begin{bmatrix} \lambda_1 & & 0 & 0 & \cdots & 0 \\ & \lambda_2 & & \vdots & & \vdots \\ & & \ddots & \vdots & & \vdots \\ 0 & & & \lambda_m & 0 & \cdots & 0 \end{bmatrix}, \quad (\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m > 0) \quad (13)$$

where the dimension of the vector $\{b\}$ and $\{x\}$ are expressed as m and n , respectively. Because of this, the size of the matrix $[A]$ or $[S]$ is $m \times n$.

The value of the Tikhonov regularization parameter α is obtained by L-curve method (Hansen, 1992). The Tikhonov regularization is workable regardless of whether all boundary conditions of Eq. (1) are prepared. If the value of α is set as zero and the all boundary conditions are prepared, the solution of Eq. (9) is equivalent to the solution obtained by directly solving Eq. (1). On the other hand, if the value of α is set as zero, the solution of Eq. (9) is the least squares solution of Eq. (6) when $m > n$, and the solution of Eq. (9) is the least norm solution of Eq. (6) when $m < n$.

2.3. Estimation procedure

All matrices originated from the total stiffness matrix $[K]$ are known if the explicit method that substitutes the value of stress before each step into the matrix $[K]$ is utilized. Therefore the total stiffness matrix $[K]$ at the initial step is calculated by the explicit method at first in the proposed method. Then the matrices $[A]$, $[A_{MF}]$, $[K_{RF}]$ are obtained from the calculated matrix $[K]$. On the other hand, the vector $\{b\}$ is calculated by using the known boundary conditions $\{du_M\}$, $\{du_F\}$ and $\{df_F\}$. By using the matrix $[A]$, the vector $\{b\}$ and Eq. (10), the solution of the vector $\{x\}$ is calculated. After calculations of stress, strain and yield judgment of each element by using the solution, the estimation process progresses to the next step. At the next step, the same calculation as the initial step is iterated. The support reaction on the face sheet is estimated by accumulating the solution of the vector $\{x\}$.

3. Verification by numerical simulation

3.1. Condition of numerical simulation

The axially symmetric models as shown in Fig. 1 are utilized in the numerical simulations. According to the result of the local compressive test by using the honeycomb core sandwich panel (Kishimoto, et al., 2015b), only the neighborhood domain of the honeycomb core sandwich panel contacted with the indenter was deformed in the test. Therefore the diameter of the analytical domain is defined as 140 mm in this numerical simulation. Table 1 shows the material constants of the constituent material of each model. All of the constituent materials are elastic perfectly plastic solid.

The model of the face sheet in each model simulates aluminum alloy. Model A is only the face sheet where the diameter is 140 mm and the thickness is 0.5 mm, supported by 2 points. This Model is equal to a circular plate supported by 2 points. Model B consists of the circular plate and 7 cylindrical cell walls where the minimum diameter is 20 mm, the maximum diameter is 140 mm, the height is 10 mm and the thickness is 0.05 mm. The constituent material of the cylindrical cell walls is also aluminum alloy. Model C is the sandwich panel consisted of the face sheet and the core layer which simulates aluminum foam. The core layer is assumed to be isotropic and homogeneous, and the material constants shown in Table 1 are reference values in previous researches (Ashby, et al., 2000; Xu, et al., 2014). In Model B and C, the delamination behavior between the face sheet and the core layer is ignored. The bonding states in those models are

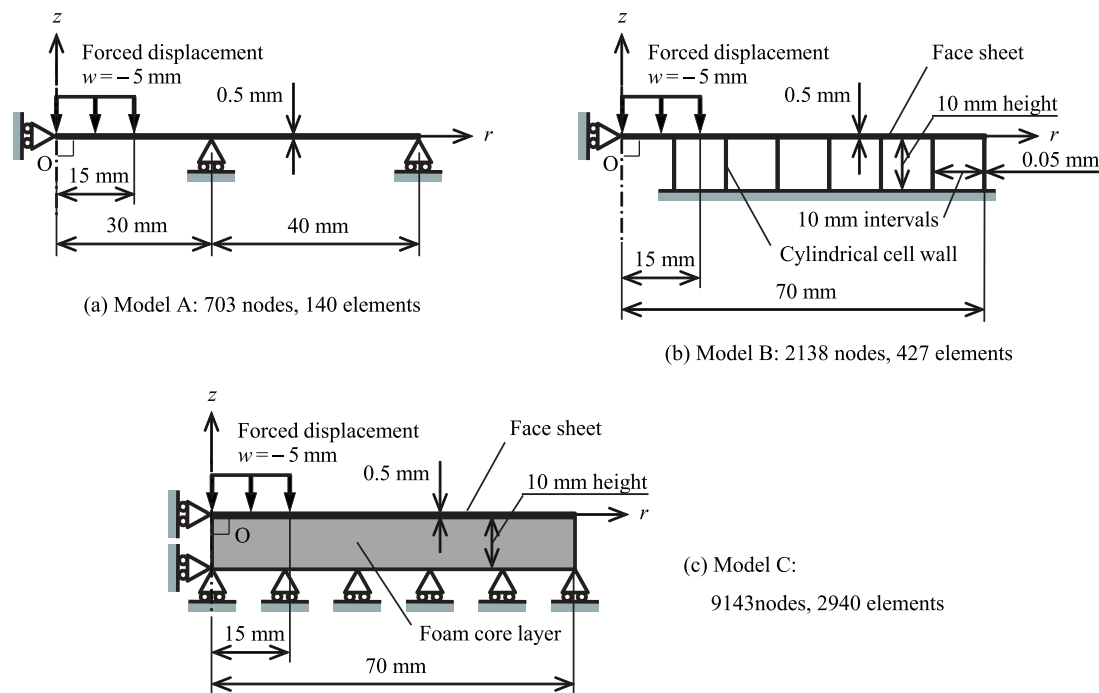


Fig. 1 Axially symmetric models for numerical simulations. Model A is only the face sheet supported by 2 points. Model B is composed of the face sheet and 7 cylindrical cell walls. Model C is composed of the face sheet and the foam core layer.

Table 1 Constituent materials of simulation models

Member	Material constant	Model A	Model B	Model C
Face sheet	Young's modulus, E		70 GPa	
	Poisson's ratio, ν		0.33	
	Yield stress, σ_y		90 MPa	
Core layer	Young's modulus, E	-	70 GPa	60 MPa
	Poisson's ratio, ν	-		0.33
	Yield stress, σ_y	-	90 MPa	1.2 MPa

simplified and the same node is configured for the nodes of the face sheet side and the core layer side on the same location in the finite element discretization.

The cylindrical coordinate system whose z -axis is the central axis of each model is defined as shown in Fig. 1. The origin is defined at the point located on the center and the bottom of each face sheet. The displacements of r -direction and z -direction are expressed as u and w , respectively. The side of each model on $r = 0$ [mm] is always located on z -axis ($u = 0$ [mm]), i.e. the side of each face sheet on z -axis is maintained to be vertical to r -axis. In Model A, only the z -direction displacement of the nodes located at the support points is fixed. The r -direction and z -direction displacements of the bottom of the cell walls in Model B ($z = -10$ [mm]) are fixed. In addition, the rotation of the bottom of the cell walls in Model B is also fixed. On the other hand, only the z -direction displacement of the nodes located on the bottom of the core layer in Model C ($z = -10$ [mm]) is fixed. Then the forced displacement $w = -5$ [mm] is applied on the center of the face sheet in each model that is 30 [mm] diameter circular area.

These models are discretized by 8-node quadrilateral element. The number of element division for each face sheet is about 140 along the r -direction and that is 1 along the z -direction. The number of element division for each cell wall in Model B is 1 along the r -direction and that is 40 along the z -direction. The number of element division for the core layer in Model C is 140 along the r -direction and that is 20 along the z -direction. Then the direct analysis is applied to obtain the displacement of each node and the nodal load of the bottom side of each face sheet (i.e. the nodal load of the nodes located on $z = 0$ [mm]).

Using the obtained z -direction displacement w as the phantom data, the inverse analysis is applied to obtain the support reaction which is equivalent to the z -direction nodal load f_z on the bottom side of the face sheet in each model. In the inverse analysis, it utilizes only the face sheet model (diameter 140 [mm], thickness 0.5 [mm]) which is the same as Model A, even if the numerical simulation by using Model B or Model C is performed. Figure 2 shows the schematic of the

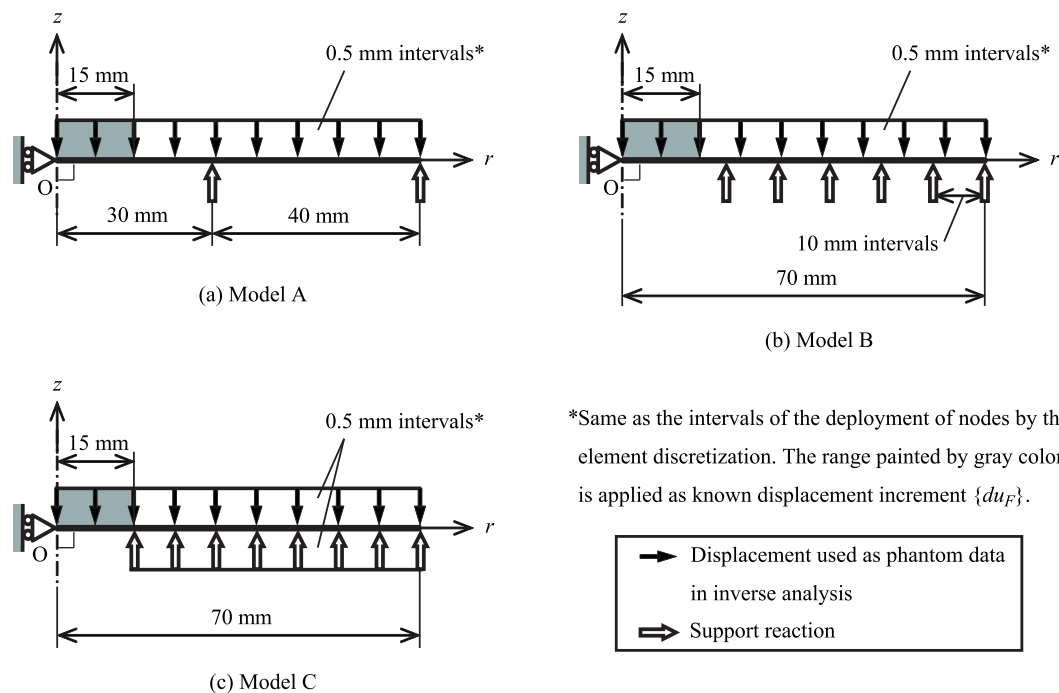


Fig. 2 Face sheet models for inverse analysis. Each of these models is consisted of 703 nodes and 140 elements.

displacement used as the phantom data and the estimated nodal load in the inverse analysis. It is defined that the location of the displacement as the phantom data is the upper side of each face sheet (i.e. the displacement of the nodes located on $z = 0.5$ [mm]). The displacement from $r = 0$ [mm] to $r = 15$ [mm] is applied as known and forced displacement, whose displacement increment is $\{du_F\}$, because it is decided by the displacement of the indenter in practical situation. The other displacement is applied as the deformed shape data including measurement error, whose displacement increment is $\{du_M\}$.

The location of the estimated nodal load is the same location of the support point, the cell wall or the core layer as shown in Fig. 2, in that the structure of the model before deformed is known. However the support reaction from $r = 0$ [mm] to $r = 15$ [mm] is assumed to be 0 N. The reason comes from that the face sheet is thin and it is difficult to distinguish the load by the force displacement and the support reaction on the same r -coordinate. These load values are evaluated as the total load on the upper side of the face sheet. In Model B, though each cell wall is consisted of 3 nodes in r -direction, the support point is represented by 1 node in the estimation of the support reaction. The increment step is 100 in all the analyses.

3.2. Results and discussion

Figure 3 shows the calculation results of the deformed shape by the direct analysis. From these figures, it seems that the direct analysis has been successfully operated because the obtained deformed shapes have no unreasonable deformed element. Especially, there are 2 buckling cell walls in Model B which are the closest to the central axis of the model, and there is also no unreasonable deformed cell wall.

Figure 4 indicates the z -direction displacement w of the upper side of each face sheet (i.e. the displacement of the nodes located at $z = 0.5$ [mm]) in the models obtained by the direct analysis. In Fig. 4, there is small difference between the displacement w of Model A and that of Model B. The cause of the small difference appeared is presumed as follows. In Model B, the 2 cell walls closest to the central axis of the model have large deformation by buckling, and the cell wall third closest to the central axis of the model (hereafter, the third cell wall for brevity) has small deformation as shown in Fig. 3. Because the support reaction by the buckling cell wall would be equal to or lower than the non-buckling cell wall, it seems that the face sheet was mainly supported by the third cell wall. The support reactions of the 4 cell walls outside the third cell wall were also lower than that of the third cell wall as discussed in detail below. Therefore it has resulted in that the location of the main support point for the face sheet in Model B was similar to that in Model A. On the other hand, there is large difference in the displacement w of Model C compared to Model A and B in the range of $r = 20$ [mm] and 40 [mm]. From this result, it can be said that the support state of the face sheet in Model C is completely different from those of Model A and B.

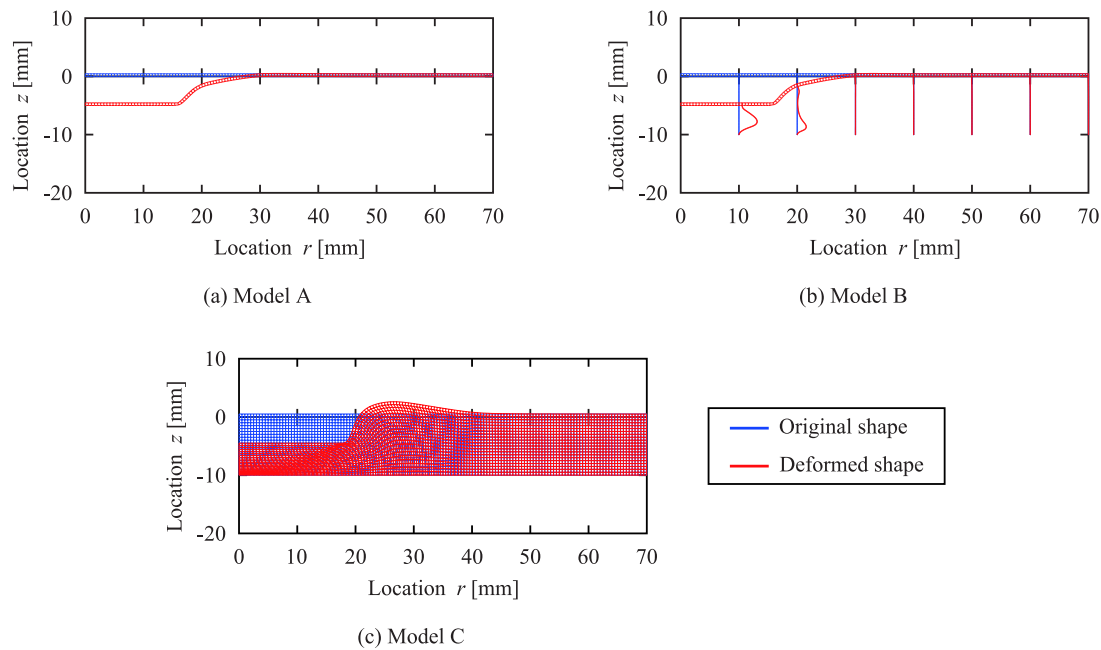


Fig. 3 Calculation results of deformed shape by the direct analysis

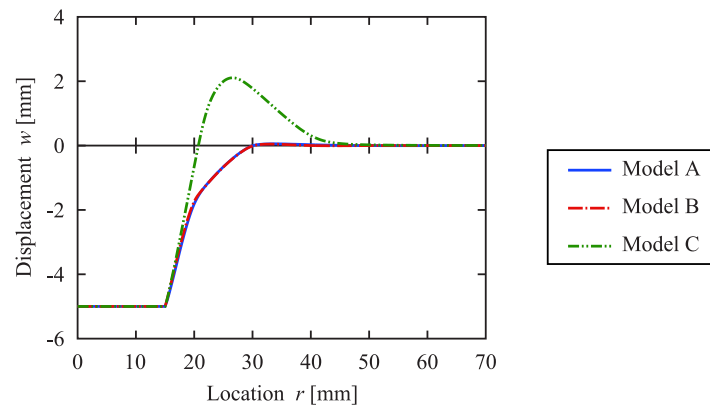


Fig. 4 Displacement of upper side of each face sheet by the direct analysis

Figure 5 and Table 2 show the calculation results of the support reaction in Model A and B. In the inverse analysis, the Tikhonov regularization parameter α was set as 0 and Eq. (10) was applied to the displacement data of Model A or B shown in Fig. 4 divided by the increment step. Then the number of decimals of each displacement data was 2 or 10 in units of millimeter, as an example. In Fig. 5 and Table 2, the estimation results by the inverse analysis have good agreement with the values calculated by the direct analysis, except $r = 10$ [mm] in Model B. Moreover it indicates that just the least squares approach enables the estimation of the support reaction in Model A and B because the Tikhonov regularization parameter α was set as 0. These results are attributed to that the row was much larger than the column concerning the size of the matrix $[A]$ and the condition number of the matrix $[A]$ was relatively small. The size of the matrix $[A]$ is 220×2 in Model A and 220×6 in Model B. The condition number of the matrix $\text{cond}[A]$ at the first increment step is 39 in Model A and 4315 in Model B, for example.

Taking the estimation results of the support reaction into account, the discussion turns to that there was small difference between the displacement w of Model A and that of Model B. From the direct analysis result of Model B as shown in Fig. 5, the support reaction of the cell wall whose location is not $r = 30$ [mm] is also nonzero. Especially, the support reaction of the cell wall at $r = 20$ [mm] is comparable to that of the third cell wall even though the cell wall at $r = 20$ [mm] is buckled as shown in Fig. 3. However it is interpreted from little difference between Model A and Model B in terms of the displacement w that the support reactions of the cell walls except the third cell wall in Model B were not so large as to restrain the face sheet from deforming in this simulation. From the perspective of the inverse analysis, the esti-

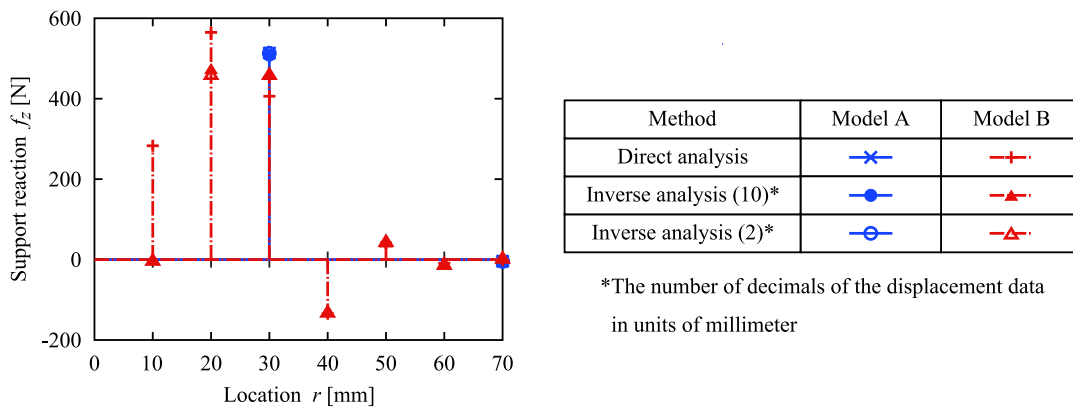


Fig. 5 Support reaction on bottom side of each face sheet. The Tikhonov regularization parameter α was set as 0 in each inverse analysis.

Table 2 Detail of support reaction as shown in Fig. 5

Model A				
Location r [mm]	Direct analysis	Inverse analysis (10)*		Inverse analysis (2)*
30	513.51	508.31	(−5.20)	512.88 (−0.62)
70	−4.05	−5.06	(−1.01)	−4.70 (−0.65)
Model B				
Location r [mm]	Direct analysis	Inverse analysis (10)*		Inverse analysis (2)*
10	282.92	0.00	(−282.92)	0.00 (−282.92)
20	564.83	472.07	(−92.75)	459.46 (−105.37)
30	406.12	460.02	(+53.90)	459.45 (+53.33)
40	−134.41	−130.17	(+4.24)	−131.55 (+2.86)
50	41.36	41.00	(−0.35)	44.15 (+2.80)
60	−8.93	−9.50	(−0.57)	−12.85 (−3.93)
70	−1.30	1.44	(+2.73)	2.71 (+4.01)

Indicated value: Support reaction [N] (Difference from the direct analysis [N])

*The number of decimals of the displacement data in units of millimeter

mated support reaction would be similar to that of Model B if the deformed data of Model A was utilized and supporting points were set at the same locations as those of Model B. This is caused by that the proposed method selects the solution minimizing the evaluation function expressed as Eq. (9). Based on the minimization of the first term of Eq. (9), i.e. the residual sum of squares of the observation equation (Eq. (6)), the load that is most likely to produce the deformed shape data is distributed to the supporting points given in advance. Therefore it is concluded that the proposed method requires the location of the support point as a priori information.

Figure 6 shows the calculation results of the support reaction on the face sheet in Model C. In the inverse analysis, the size of the matrix $[A]$ is 220×220 . The condition number of the matrix $\text{cond}[A]$ at the first increment step is 2.01×10^{11} , for example. The Tikhonov regularization parameter α was set as 0 and Eq. (10) was applied to the displacement data of Model C shown in Fig. 4 divided by the increment step. Then the number of decimals of each displacement data was 2 or 10 in units of millimeter, as an example. The results show that the estimated support reactions have oscillatory components and the estimation has resulted in failure.

Because the estimation when the Tikhonov regularization parameter $\alpha = 0$, that is equivalent to the least squares estimation, has been unable to obtain reasonable solution, the value of α was selected by L-curve. Figure 7 shows the L-curve at the first increment step in the inverse analysis process, as an example. In Fig. 7, the first term of Eq. (9) which means the residual sum of squares of the observation equation (Eq. (6)) increases and the second term decreases with the increase in the Tikhonov regularization parameter α . Furthermore, when the Tikhonov regularization parameter $\alpha = 2.14 \times 10^{-11}$, the L-curve visually leaves from the vertical axis, which is the line of $(\{b\} - [A]\{x\})^T (\{b\} - [A]\{x\}) = 0$ and the curvature of the L-curve is maximized.

Using the feature of the L-curve as identified above, the value of α maximizing the curvature of the L-curve was selected at each increment step in the inverse analysis and the support reaction on the face sheet was estimated again. The estimation results are shown in Fig. 8. In Fig. 8, the estimated support reactions have good agreement with the distribution calculated by the direct analysis from $r = 20$ [mm] to $r = 35$ [mm] and from $r = 55$ [mm] to $r = 70$ [mm], regardless of

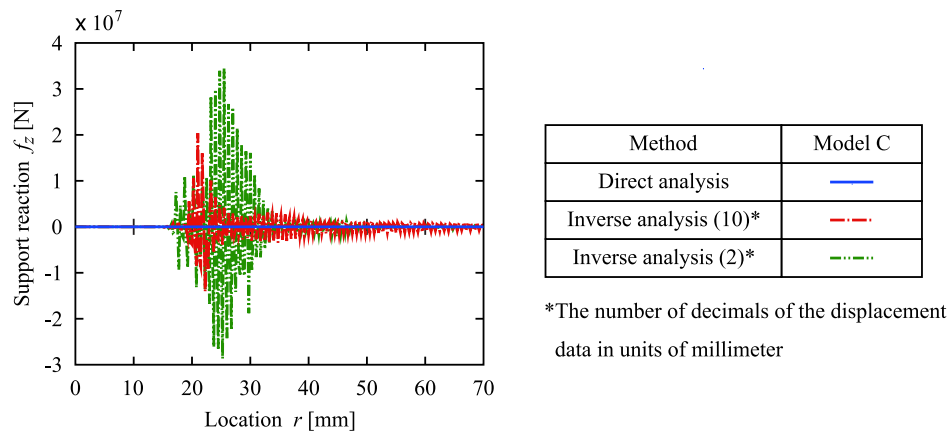


Fig. 6 Support reaction on bottom side of face sheet in Model C. The Tikhonov regularization parameter α was set as 0 in the inverse analysis.

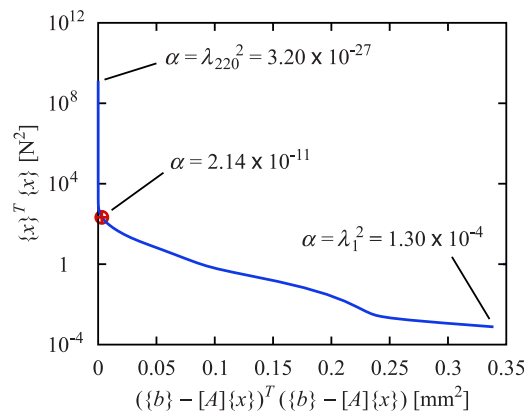


Fig. 7 Example of L-curve when varying Tikhonov regularization parameter α . The figure shows L-curve from $\alpha = \lambda_{220}^2$ (λ_{220} : the minimum singular value of the matrix $[A]$) to $\alpha = \lambda_1^2$ (λ_1 : the maximum singular value of the matrix $[A]$) at the first increment step. From the L-curve, $\alpha = 2.14 \times 10^{-11}$ was selected.

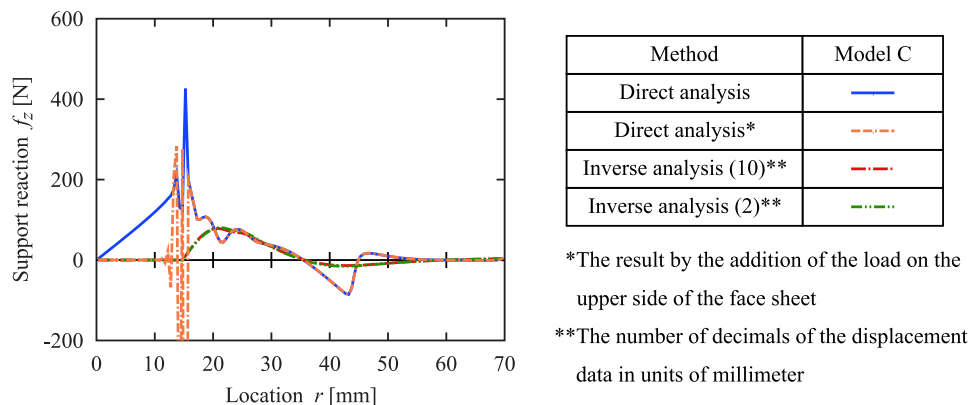


Fig. 8 Support reaction on bottom side of each face sheet. The Tikhonov regularization parameter α was selected by L-curve at each increment step in the inverse analysis.

the number of decimals of the displacement data. Though the support reactions from $r = 15$ [mm] to $r = 20$ [mm] and from $r = 35$ [mm] to $r = 55$ [mm] have been oversmoothed in this estimation, those tendency could be derived. In the calculation result by the direct analysis, the sum of the support reaction and the load on the upper side of the face sheet (i.e. the nodal load of the nodes located on $z = 0.5$ [mm]) is approximately zero from $r = 0$ [mm] to $r = 15$, though the sum is unstable near $r = 15$ where the boundary condition by the forced load is noncontiguous. Therefore bearing in mind that the proposed method could derive the tendency of the support reaction from $r = 15$ [mm] to $r = 70$ [mm], it seems

that the assumption for the support reaction from $r = 0$ [mm] to $r = 15$ [mm] is functional with respect to the estimation except for the case that the support reaction directly below the point of the external load is necessary. The further study including enhancement of the estimation accuracy of the proposed method is an issue in the near future.

4. Conclusion

In this study, we have developed a novel method to estimate the support reaction on the face sheet of the sandwich panel by the inverse analysis of the deformed shape data of the face sheet. The proposed method consists of the finite element method based on strain increment theory and the Tikhonov regularization whose regularization parameter is selected by L-curve method. In order to demonstrate the validity of the proposed method, numerical simulations have been performed. From the simulation results, the followings were summarized.

- (1) The distributions of the support reaction estimated by the proposed method have good agreement with the calculation results by the direct analysis outside the area directly below the external load.
- (2) The proposed method requires the location of the support point as a priori information.
- (3) In the inverse analysis, it seems that the least squares approach can be workable when the condition number of the matrix in the observation equation is relatively small. On the other hand, the Tikhonov regularization should be applied to smooth the solution when the condition number is large.
- (4) The case that the Tikhonov regularization derives oversmoothed distribution in terms of the estimation of the support reaction is observed. A further study of enhancement of the estimation accuracy of the proposed method should be conducted.

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