

Prediction of the friction coefficient of filled rubber sliding on dry and wet surfaces with self-affine large roughness

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Abstract

The friction of filled rubber on a rough surface is mainly determined by the rubber viscoelasticity and the surface property of multiple-scale asperities that can be represented by the power spectral density of the surface profile (i.e., power spectrum of surface roughness). This paper investigates a prediction model of rubber friction on dry and wet surfaces with large roughness under lightly squeezing, and finds a high stationary friction coefficient that depends on sliding speed. To this end, we demonstrated friction testing at low velocities with carbon-black-filled rubber and a hard substrate having self-affine surface roughness. From the experiment results, we estimated the hysteresis friction coefficient related to energy dissipation resulting from cyclic deformations of the viscoelastic rubber by applying the theory developed by Persson [(J. Chem. Phys. **115**, 3840 (2001))]. We discussed the additional factor, an adhesion force, which also increases the friction coefficient. We concluded that the hysteresis loss of rubber viscoelastic deformation contributes most of the friction force, accounting for the nonlinear viscoelastic behavior of filled rubber, and that the operative surface wavelength extends to the order of micrometers.

Key words : Rubber friction, Viscoelasticity, Power spectrum of surface roughness, Self-affine fractal, Friction test

1. Introduction

Tire rubber is the sole mechanical component of a vehicle that is in contact with the road and sustains the weight of the vehicle. From a mechanical viewpoint, the friction coefficient at the interface between the rubber and road is a prime factor determining the driving performance of a vehicle in terms of braking, operability and environmental friendliness. It is thus important that engineers understand the dependence of rubber-road friction on the viscoelasticity of the rubber and surface characteristics of the road in the overall design of a tire from the material development of rubber compounds to the patterning of tire treads.

Finite element analysis (FEA) can give an approximate solution to the continuum behaviors of the contact mechanics and frictional system of a rubber material. However, on a microscale, there are remaining issues such as the real contact area accounting for repulsive and attractive interactions. For example, the contact between an elastic material and a rough substrate such as a self-affine surface have been extensively studied in FEAs (Hyun et al, 2004, Pei et al, 2005, Hyun et al, 2007) and molecular dynamics simulations (Yang et al., 2006, Yang and Persson, 2008, Pastewka and Robbins, 2014). Furthermore, there remain considerable temporal and spatial challenges in elucidating the comprehensive friction mechanism of rubber sliding over surface asperities of multiple sizes employing such numerical procedures (Mo et al, 2009, Pálfi et al., 2012).

Meanwhile, two theoretical models of rubber friction on dry and rough surfaces have been developed by Klüppel and Heinrich (2000) and by Persson (2001), who proposed a hysteresis friction coefficient relating to the viscoelastic deformation of rubber along self-affine asperities in different ways. In the past decade, the two models and their extensions with adhesion effects have been experimentally validated for several combinations of rubber materials and substrates; e.g., unfilled or filled rubber materials and dry or wet surfaces (Westermann et al., 2004, Gal et al., 2005, Gal and Klüppel, 2008, Lorenz et al., 2011, Lorenz et al., 2013a). Furthermore, two new theories are being applied in developing a practical model for rubber friction in tire traction (Heinrich and Klüppel, 2008) and tire body dynamics (Persson, 2011). However, most of these related studies have been still performed by confined research groups, e.g., (Busse et al., 2010, Lorenz et al., 2013b, Scaraggi and Persson, 2014), and exhaustive information is required to construct a common framework for rubber-road friction. In this context, our study proposes such a concept of the friction mechanism to engineers widely, providing measurements and predictions for rubber friction under conditions different from those of previous studies (Westermann et al., 2004, Gal et al., 2005, Gal and Klüppel, 2008, Lorenz et al., 2011, Lorenz et al., 2013a).

In pioneering work by Grosch (1963), the author stated that rubber friction on a rough surface, which arises from the interfacial adhesion and hysteresis deformation of the rubber, depends on velocity and temperature, and that the obtained master curves of the friction coefficients are strongly correlated to the viscoelastic properties of the rubber. In our experiment, as seen in Fig. 1, the friction of filled rubber on a rough surface is measured with and without lubrication at low sliding velocities. While the friction coefficient for a dry surface increases with increasing velocity (solid line), it remains constant for the lubricated surface (dashed line). One possible reason for this result is that the hard additive particles in the lubricant oil, which were organomolybdenum compounds in this case, infilled the surficial cavities of microscopic size, and prevented the rubber from penetrating the cavities, which consequently weakened the hysteretic behavior.

The results in Fig. 1 raise the question of how the hysteresis loss of the rubber sliding on the rough surface contributes to the frictional force, or in other words, we wish to know in which operative range of the corrugation sizes does the rubber undergo hysteretic deformation along the sliding surface. The two new theories described previously (Klüppel and Heinrich, 2000, Persson, 2001) provide good prediction models that address the question. Experimental validations of the two models have shown fair agreement between theory and measurements of the friction coefficient of carbon-black-filled rubber sliding on asphalt (Westermann et al., 2004, Gal et al., 2005). We here adopt the rubber friction model developed by Persson (2001) because we can explicitly give the required range of frequencies for the calculation, which corresponds to the operative range of the corrugation sizes causing the hysteresis loss. To compare with the Persson model, we carry out a friction test using carbon-black-filled rubber and dry and wet substrates with large roughness under a low load condition in the normal direction. The adhesion force for rubber friction is in general proportional to the true contact area, and we thus select a weak normal force to weaken the adhesive effect on the total friction force.

The remainder of the paper is organized as follows. In Section 2, we review Persson's friction theory (Persson,

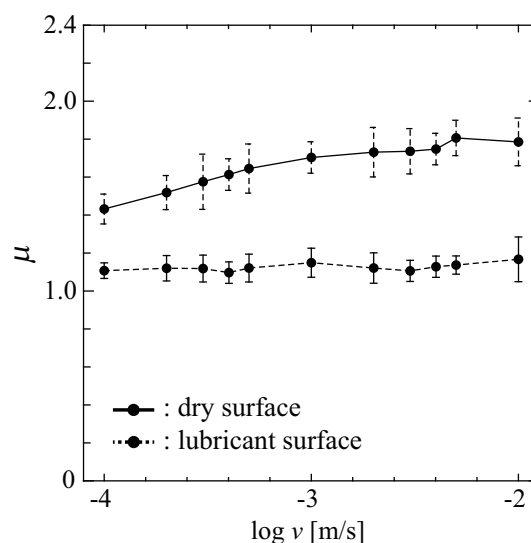


Fig. 1 Correlations of rubber friction on a rough surface under dry and lubricated conditions at room temperature. We greased the same rough surface with lubricant oil including organomolybdenum compound additives. The dimensions of the rubber block were $20 \times 20 \times 4 \text{ mm}^3$ and the normal load stress was 2450 Pa.

2001, Persson et al., 2005a) and describe the key equations required for the calculation of the hysteresis friction coefficient. In Section 3, we explain the materials we employ in the friction test. We first provide surface information, measuring the power spectrum of surface roughness. We next describe the carbon-black-filled rubber compound and identify its complex elastic modulus with a generalized Maxwell model. In Section 4, we conduct a friction test and assess the numerical hysteresis part of the friction coefficient for comparison with the measured friction coefficient. In Section 5, we discuss the additional factor as an adhesive shear force, which increases the friction coefficient. Section 6 presents the summary and conclusion.

2. Mathematical descriptions of Persson's friction theory

2.1. Power spectrum of surface roughness with a self-affine fractal

Considering a rough surface with asperities at multiple scales, the power spectrum of surface roughness provides good guidance for the wide-scale surface characteristic in wavenumber space. The power spectrum of surface roughness $C(q)$ was defined by Nayak (1971) as

$$C(q) = \frac{1}{(2\pi)^2} \int d^2x \langle h(\mathbf{x})h(\mathbf{0}) \rangle e^{-i\mathbf{q} \cdot \mathbf{x}}, \quad (1)$$

where $z = h(\mathbf{x}) \in \mathbb{R}$ is the substrate height at position $\mathbf{x} \in \mathbb{R}^2$, measured from the average surface plane, and $\mathbf{q} \in \mathbb{R}^2$ is a wavevector. Furthermore, $\langle \cdot \rangle$ indicates ensemble averaging. Assuming that the surface profile has inplane isotropy, we only treat the amplitude of the wavevector as the wavenumber $q = |\mathbf{q}|$. We also assume that the surface is a self-affine fractal with fractal dimension D_f in some wavenumber region $[q_0, q_1]$. If $q_0 \ll q_1$, Equation (1) can be approximated with the exponent $H (= 3 - D_f)$ in the following form (Persson, 2001):

$$C(q) \approx \frac{H}{2\pi} \left(\frac{h_0}{q_0} \right)^2 \left(\frac{q}{q_0} \right)^{-2(H+1)}. \quad (2)$$

Here $\langle h^2 \rangle = h_0^2/2$. Equation (2) shows that a self-affine surface follows a power law described by $C(q) \sim q^{-2(H+1)}$.

It is known that many surfaces tend to be a nearly self-affine fractal. In our experiment, the surface specimen indeed exhibits a feature similar to a power law as we will mention in Section 3.1. Figure 2(a) illustrates the surface roughness power spectrum for a typical rough surface. The log-log plot of $C(q)$ has two types of wavenumber regions. One is a macroscopic area, referred to as Region I and corresponding to the range $[q_L, q_0]$ where $q_L \approx 2\pi/L$ and L is the characteristic length of a specimen. The arithmetic average roughness (Ra) is mainly determined by the surface profile in Region I. We refer to the next range of $[q_0, q_1]$ as Region II; these wavenumbers are referred to as the lower cut-off and upper cut-off wavenumbers, respectively. The surface property in Region II is a self-affine fractal represented by Eq. (2).

2.2. Rubber friction model of Persson's theory

When squeezing and sliding rubber as a hyperelastic material on a rigid and rough surface, the rubber cyclically deforms along the surface corrugations with different wavelengths and amplitudes at each magnification ζ , as shown in

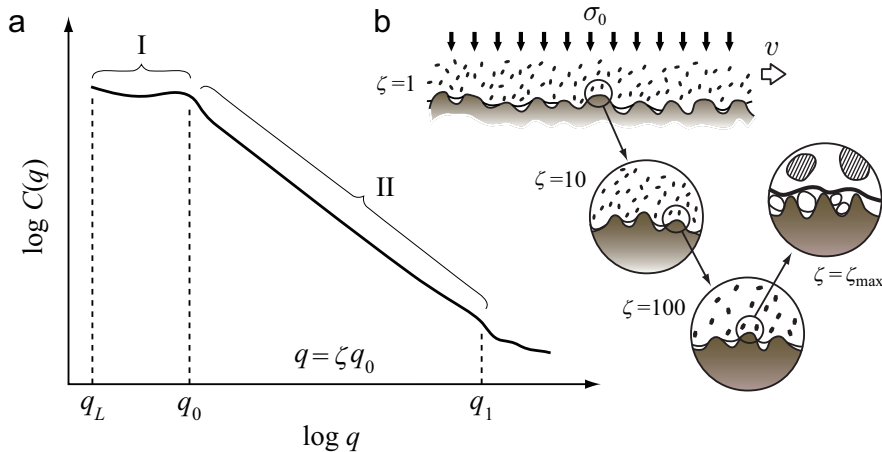


Fig. 2 (a) A schematic drawing of the power spectrum of surface roughness for a conventional rough surface; (b) a conceptual sketch of the mechanism of rubber friction for a rubber block sliding on multiple-scale asperities that form a self-affine fractal surface, following Persson's idea (Persson, 2001, Persson et al., 2005a).

Fig. 2(b). In this context, Persson developed rubber-friction theory to predict the friction coefficient due to pure hysteresis of rubber deformation without adhesion (Persson, 2001). In the theory, let A_0 and $A(q)$ be the macroscopic (nominal) contact area and the contact area at some wavenumber scale, respectively, the relative area of real contact is

$$\frac{A(q)}{A_0} = \frac{2}{\pi} \int_0^\infty dx \frac{\sin x}{x} \exp[-x^2 G(q)],$$

$$\simeq (1 + [\pi G(q)]^{3/2})^{-1/3}. \quad (3)$$

In Eq. (3), $G(q)$ is

$$G(q) = \frac{1}{8} \int_{q_0}^q dq q^3 C(q) \int_0^{2\pi} d\phi \left| \frac{E(qv \cos \phi)}{(1 - \nu^2)\sigma_0} \right|^2, \quad (4)$$

where v , σ_0 and ν are the sliding velocity, nominal stress and Poisson's ratio of the rubber, respectively. Furthermore, $E(qv \cos \phi) = E(\omega) \in \mathbb{C}$, which is the complex elastic modulus as a function of the angular frequency ω . With the three elements mentioned above, namely $C(q)$, $A(q)$ and $E(\omega)$, the hysteresis friction coefficient μ_h can be expressed by

$$\mu_h = \frac{1}{2} \int_{q_0}^{q_1} dq q^3 C(q) \frac{A(q)}{A_0} \int_0^{2\pi} d\phi \cos \phi \operatorname{Im} \frac{E(\omega)}{(1 - \nu^2)\sigma_0}. \quad (5)$$

In the case of $\sigma_0 \ll E(0)$, $G(q) \gg 1$ for most q -values of interest, and Eq. (3) can be written in the simpler form

$$\frac{A(q)}{A_0} \simeq \frac{2}{\pi} \int_0^\infty dx \exp[-x^2 G(q)] = [\pi G(q)]^{-1/2}, \quad (6)$$

where $\sin x \simeq x$. Substituting Eq. (6) into Eq. (5), σ_0 vanishes, and μ thus does not depend on σ_0 . Furthermore, Eq. (5) reveals that μ depends on q and not on the magnitude of the complex elastic modulus, because $A/A_0 \propto \alpha^{-1}$ as $E \rightarrow \alpha E$, where α is a scaling parameter.

Taking into account the self-affine fractal surface, Eq. (5) can be approximated using Eqs. (2)–(4) as

$$\mu_h \simeq \frac{1}{4\pi} (q_0 h_0)^2 H \int_1^{q_1/q_0} d\zeta \zeta^{-2H+1} \left(1 + [\pi G(\zeta)]^{3/2}\right)^{-1/3} \int_0^{2\pi} d\phi \cos \phi \operatorname{Im} \frac{E(\zeta q_0 v \cos \phi)}{(1 - \nu^2)\sigma_0}, \quad (7)$$

where $G(\zeta)$ is

$$G(\zeta) = \frac{1}{16\pi} (q_0 h_0)^2 H \int_1^{q_1/q_0} d\zeta \zeta^{-2H+1} \int_0^{2\pi} d\phi \left| \frac{E(\zeta q_0 v \cos \phi)}{(1 - \nu^2)\sigma_0} \right|^2. \quad (8)$$

Note that q is replaced with ζq_0 , recalling ζ is the spatial magnification as shown in Figs. 2(a) and (b).

The fundamental idea in Persson's theory is that the macroscopic work resulting from the shear friction force is equivalent to the total dissipative energy built up from the hysteresis loss due to cyclic deformations of the interfacial rubber across the surface at multiple scales. However, it is hard to accept that such a purely hysteretic behavior has a nanometer feature because it becomes dominated by other phenomena such as an adhesion interaction, contamination interference and inhomogeneous deformation of filled rubber at the microscale. Therefore, an upper cut-off wavenumber should be considered as a free parameter that cannot be determined only by surface information. Persson clearly states that an upper cut-off wavenumber may be related not only to surface properties but also rubber properties (Persson et al., 2005a), and his group indeed determined q_1 for the sizes of compact rubber-wear particles in a friction experiment (Lorenz et al., 2011). In their study, it was estimated that $q_1 = 5 \times 10^6 \text{ m}^{-1}$, which was consistent with the diameter of 20 μm of the smallest wear particle. Our prime interest as regards Eq. (7) is how to adequately determine the free parameter q_1 . In this regard, we will give a qualitative assessment of q_1 with respect to the rate of increase in the friction coefficient obtained from the measurements and simulations in Section 4.2.

3. Materials

3.1. Measurement of the power spectrum of surface roughness

We employed commercial anti-slip tape (Safety Walk type ATM, 3M Japan Ltd.) as a hard and rough surface. With two types of laser microscopes, we measured the surface profiles at the three magnifications listed in Table 1. Figures 3(a)–(c) show the typical surface profiles and the probability distribution of the surface height, measured by microscopes at 40- and 200-power magnification. Note that the raw data for 200-power magnification are contaminated by much optical noise, and the measured height data were thus smoothed with a median filter. It is observed that the surface profile for 200-power magnification has a shorter wavelength than that for 40-power magnification. Figure 4 shows the

Table 1 Conditions of the surface measurements.

instrument (manufacturer)	microscope VR-3100 (Keyence Co., Ltd.)	laser microscope VK-9500 (Keyence Co., Ltd.)	
height resolution	0.1 μm	0.01 μm	
XY calibration	7.403 μm	1.391 μm	0.137 μm
magnification	$\times 40$	$\times 200$	$\times 2000$
observed area	15.16 $\times$ 15.16 mm^2	2.849 $\times$ 2.849 mm^2	0.281 $\times$ 0.281 mm^2
measuring points	2048 $\times$ 2048	2048 $\times$ 2048	2048 $\times$ 2048
the num. of samples	2	3	6

Table 2 Measured surface properties of a road specimen.

name	Safety-Walk type A TM (3M Japan Ltd.)
h_0 [μm]	82.89
q_0 [1/m]	5070
b [m^4]	-16.57 ± 0.19
H	0.631 ± 0.014

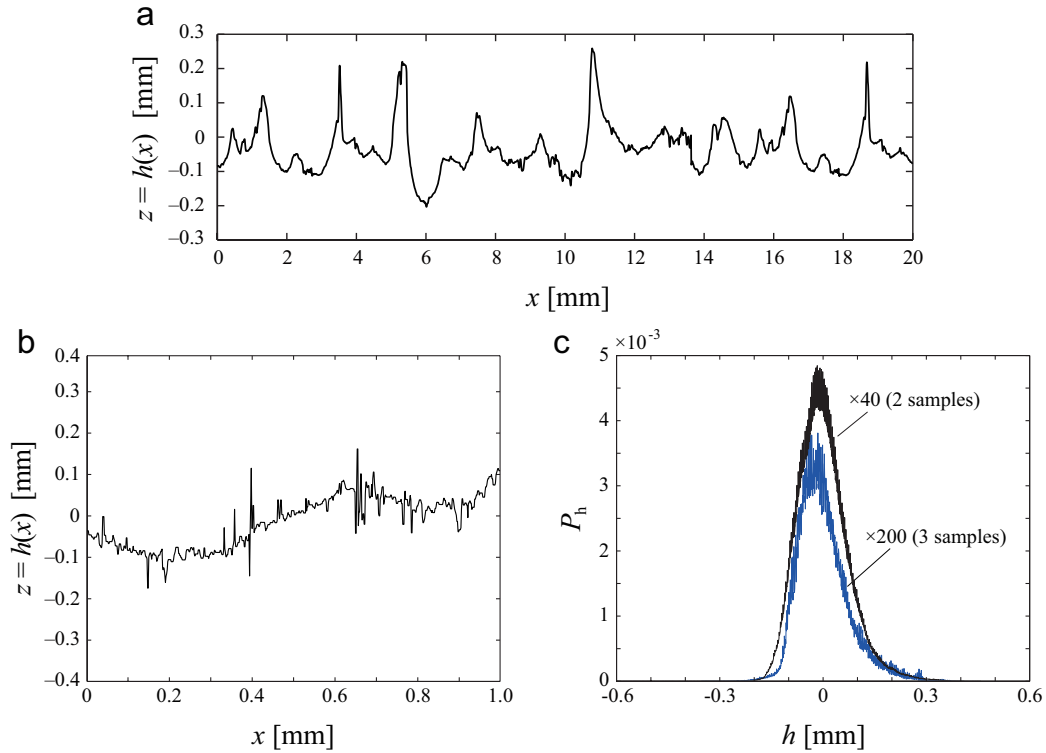


Fig. 3 Typical surface profiles measured by the two microscopes with (a) 40- and (b) 200-power magnifications. Note that the scales on the two axes in (a) are unequal and that the measured height data of (b) are smoothed using a median filter. (c) Probability distributions of the surface height.

calculated power spectrum of surface roughness for a magnifying power of 40, 200 or 2000. Note that we conducted an averaging procedure for each $C(q)$ and that we ignored the high-frequency components resulting from optical noise at each magnification. The calculated $C(q)$ well represents the self-affine fractal property. The scale of the measured minimum wavelength is on the order of a few micron, corresponding to the maximum wavenumber, $\sim 10^6$ 1/m, which is large enough to predict the experimental friction coefficients at the measured range of sliding velocities and temperatures (see Section 4).

As shown in Fig. 2(a), $C(q)$ is divided into Regions I and II. In each of the two regions, bilinear equations are approximated for the measured $C(q)$ through linear least squares fitting:

$$\begin{cases} \log C(q) = b, & q_L \leq q \leq q_0, \\ \log C(q) - b = -2(H+1)(\log q - \log q_0), & q_0 \leq q, \end{cases} \quad (9)$$

where b is a constant as illustrated in the inset of Fig. 4. Table 2 gives the obtained surface property of the specimen.

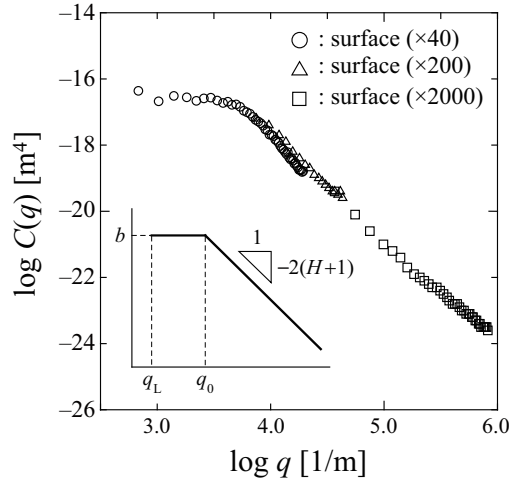


Fig. 4 Power spectra of surface roughness for 40-, 200- and 2000-power magnifications.

Here h_0 is calculated using Eqs. (2) and (9) as

$$h_0 = \sqrt{\frac{2\pi}{H}} q_0^2 10^b \approx 82.886 \mu\text{m}. \quad (10)$$

From the raw surface-height data, we can confirm that $h_0 = \sqrt{2\langle h^2 \rangle} \approx 94.877 \mu\text{m}$, which is of the same order as the result obtained with Eq. (10), where we remove the top and bottom 1% of the height distribution data to filter noise. The surface specimen has a wavelength corresponding the lower cut-off q_0 as $\lambda_0 = 2\pi/q_0 \approx 0.813 \text{ mm}$, and the exponent $H \approx 0.63$ corresponds to the fractal dimension $D_f \approx 2.37$, which is within the allowable region of 2.2 to 2.8 that is common for self-affine fractal surfaces (Persson et al., 2005a). In particular, our specimen is categorized as a rough surface because $H > 0.7$ in the two earlier studies (Westermann et al., 2004, Lorenz et al., 2011).

3.2. Complex elastic modulus of the rubber sample

We present the relaxation stress and modulus of the rubber sample employing a generalized Maxwell model with n terms as illustrated in Fig. 5, each of which is

$$\sigma(t) = \sigma_0 + \sum_{i=1}^n \sigma_i \exp\left(-\frac{t}{\tau_i}\right), \quad E(t) = E_0 + \sum_{i=1}^n E_i \exp\left(-\frac{t}{\tau_i}\right), \quad (11)$$

where σ_i , E_i and τ_i are the stress, Young's modulus and relaxation time of the i -th term, respectively. The relaxation time has the relationship $\tau_i = \eta_i/E_i$, where η_i is the i -th coefficient of viscosity. Applying dynamic deformation to the rubber, the complex elastic modulus can be described as

$$E(\omega) = E' + iE'' = E_0 + \sum_{i=1}^n \frac{1}{\frac{1}{E_i} + \frac{i}{\omega\eta_i}}, \quad (12)$$

where E' and E'' are storage and loss elastic moduli, respectively.

For a sliding substance, we employed carbon-black-filled rubber with the compound formulation listed in Table 3. To obtain the viscoelasticity of this rubber sample, we conducted a torsional shear test using a commercial rheometer and measured the dynamic elastic modulus at several frequencies $f (= \omega/2\pi)$ and temperatures T . The dynamic strain amplitude was 0.1%. The shift factor a_T as a function of T can be expressed by the Williams–Landel–Ferry equation (Williams, 1955):

$$\log a_T = \frac{-C_1(T - T_s)}{C_2 + T - T_s}, \quad (13)$$

where T_s is a reference temperature. The employed rubber sample has a shift factor with $C_1 = 10.36$ and $C_2 = 144.5$ at $T_s = 25 \text{ }^\circ\text{C}$. Using a_T , we obtained the master curve of viscoelastic data for E' , E'' and $\tan \delta = |E''/E'|$, as shown in Fig. 6. Note that we used the isotropic relationship $G = E/2(1 + \nu)$ for $\nu = 0.5$ to convert the measured shear modulus G to E . On the basis of the measurements, we construct the fitting curves with the 20-term generalized Maxwell model of

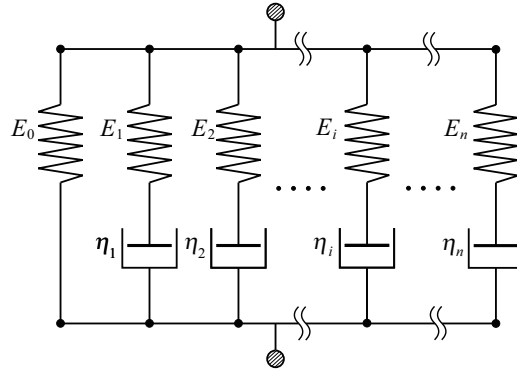


Fig. 5 Schematic of the n -term generalized Maxwell model.

Table 3 Compound formulation of a rubber sample.

SBR [phr]	100
carbon black [phr]	70
softener [phr]	40
sulfur [phr]	1.5
others: age inhibitor, accelerator, etc.	

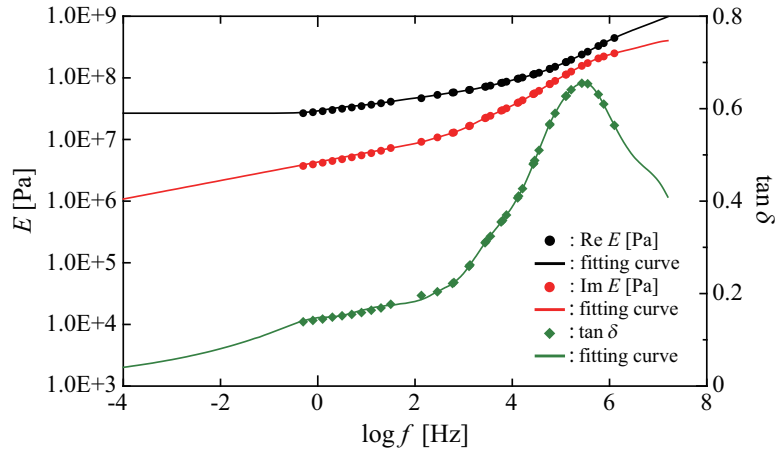


Fig. 6 Complex elastic properties of a rubber specimen: the measured data at a strain amplitude of 0.1% and a reference temperature of 25 °C, and fitting curves obtained with a 20-term generalized Maxwell model.

Eq. (12), where $E(0) = E_0 \approx 26.72$ MPa. Note that the fitted E'' is linearly interpolated in the lower frequency region to prevent undulation due to the higher-order terms of the generalized Maxwell model.

It is well known that the storage elastic modulus E' of filled rubber depends on the strain amplitude, which is so-called nonlinear viscoelastic behavior. The storage elastic modulus in general is lower under large strain (Payne, 1964, Payne, 1965, Westermann et al., 2004, Lorenz et al., 2013a). As we mentioned in Section 2.2, the friction coefficient of Persson's theory does not depend on $|E|$ in the case of $\sigma_0 \ll E(0)$. However, it is unclear how μ depends on the relative change in the amplitudes between E' and E'' . We here modify the storage elastic modulus as $\beta E'$ to concisely represent nonlinear viscoelastic behavior with a scaling parameter, $0 \leq \beta \leq 1$. The corrective complex elastic modulus is then written as $E = \beta E' (1 + i \tan \delta / \beta)$. When $\sigma_0 \ll E(0)$, Eqs. (4) and (6) with $\beta E'$ are rewritten as

$$\frac{A(q)}{A_0} = \frac{(1 - \nu^2)\sigma_0}{\beta} \left[\frac{\pi}{8} \int_{q_0}^q dq q^3 C(q) \int_0^{2\pi} d\phi E'^{1/2} \left(1 + \frac{\tan^2 \delta}{\beta^2} \right) \right]^{-1/2}. \quad (14)$$

For a common type of rubber, $\tan \delta$ is small at very low frequency. Hence, $A/A_0 \propto 1/\beta$ and furthermore $\mu_h \propto 1/\beta$ for $E' \rightarrow \beta E'$ when $(\tan \delta)/\beta \ll 1$ at very low frequency corresponding to lower sliding speed. It is natural that the real contact area increases as rubber becomes softer, thus increasing the friction coefficient.

4. Experimental results and numerical analysis

4.1. Friction testing

Using a tribological testing instrument (TL201Ts, Trinity-Lab Inc.), we conducted a friction test for the surface and

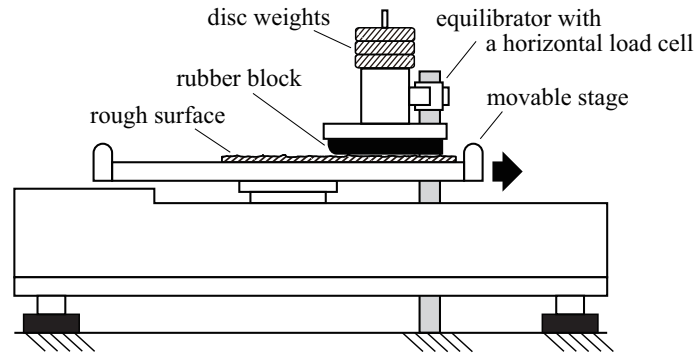


Fig. 7 Schematic of a rectilinear friction test with a tribological testing instrument (TL201Ts, Trinity-Lab Inc.).

Table 4 Conditions of friction tests for three types of rubber blocks.

rubber sample	edge treatment	block dimension (mm ³)	normal force (g)	sliding-velocity (mm/s)	sliding distance (mm)	trials
R100	no	20.0 × 20.0 × 4.0	100	0.1 ~ 1.0	20.00	3
R100'	yes	20.0 × 20.0 × 4.0	100	0.1 ~ 1.0	20.00	3
R200	no	28.3 × 28.3 × 4.0	200	0.1 ~ 1.0	20.00	3
R200'	yes	28.3 × 28.3 × 4.0	200	0.1 ~ 1.0	20.00	3

rubber specimens described in Section 3. Figure 7 is a schematic of the friction test; the stage with the rough surface can linearly move in contact with a fixed rubber block. While sliding the rubber block, a normal force was applied to the rubber by weighting equipment; i.e., the friction test was performed under load control. Table 4 gives the experimental conditions. We prepared two rubber blocks (R100 and R200) having different dimensions of contact area, and provided a load force, 100 or 200 g, to each rubber block so that the nominal stress was $\sigma_0 = 2450$ Pa in each case. Note that the friction coefficient of rubber is substantially independent of such low weight (Grosch, 1963). We also chamfered the front edge on the contact side of additional rubber blocks to compare the friction factor of the rubber with and without edge treatment. We refer to the two additional rubber blocks with edge treatment as R100' and R200'. To eliminate the effects of other phenomena such as an increase in the interfacial temperature, which is the so-called *flash temperature* (Persson, 2006), and stick-slip vibration over a sliding surface, which reduce μ , we first selected a range of low sliding velocities from 0.1 to 1.0 mm/s. In this friction test, μ was obtained as the ratio of shear to normal forces, where the shear force was the average of the steady-state forces measured by a horizontal load cell as illustrated in Fig. 7.

Figures 8(a) and (b) show the measured friction coefficient as a function of the logarithm (base 10) of the sliding velocity for rubber blocks (a) R100 and R100' and (b) R200 and R200' with dry surfaces. Note that each of the friction coefficients in these plots is an average taken over three trials and that the range of these sliding velocities are shifted using Eq. (13) at a reference temperature of 25 °C. Comparing the results obtained with and without edge treatment, it is found that the friction coefficient of edge-treated rubber was higher at all sliding velocities because the treatment prevented the front edge of the rubber block from sticking to the surface excitations. Conversely, μ was similar for R100 and R200 and for R100' and R200', and the nominal contact area was thus large enough to present the reproducible contact of surface asperities. Hence, the nominal stress σ_0 can be applied to the framework of Persson's theory. In Figs. 8(a) and (b), each error bar shows the average standard deviation of μ taken over the three trials. Figures 8(c)–(f) show the measured μ of R100, R200, R100' and R200' for each trial, respectively. Note that the three trials correspond to three different surface specimens under dry conditions. The magnitude of μ thus depends on a geometrical factor such as the macroscopic surface variation and/or the chamfered edge profile. However, the rate of the monotone increase in μ was similar among the three curves. In the next section, we compare μ_h derived from Persson's theory with the average friction coefficient of R200' because the standard deviations were smallest for this rubber block; the standard deviation was less than 5% in each trial. The increment of $\mu(v)$ for R200' is described as $\Delta\mu = \mu(v_{\max}) - \mu(v_{\min}) \approx 0.1507$, where $v_{\min} \approx 1.496 \times 10^{-4}$ m/s and $v_{\max} \approx 1.496 \times 10^{-3}$ m/s.

We next performed the additional friction tests for R200' under both dry and wet conditions at the wider range of sliding velocities from 0.1 to 10 mm/s, where we prepared the wet surface to mist with pure water. Figure 9 shows the average of measured friction coefficients on dry and wet surfaces for three trials as a function of the logarithm (base 10) of the sliding velocity. The wet surface lubricated with water results in the drop in the friction coefficient throughout the measuring velocities. Thus, the adhesion effects were removed at the wet interfaces and we assume that the difference

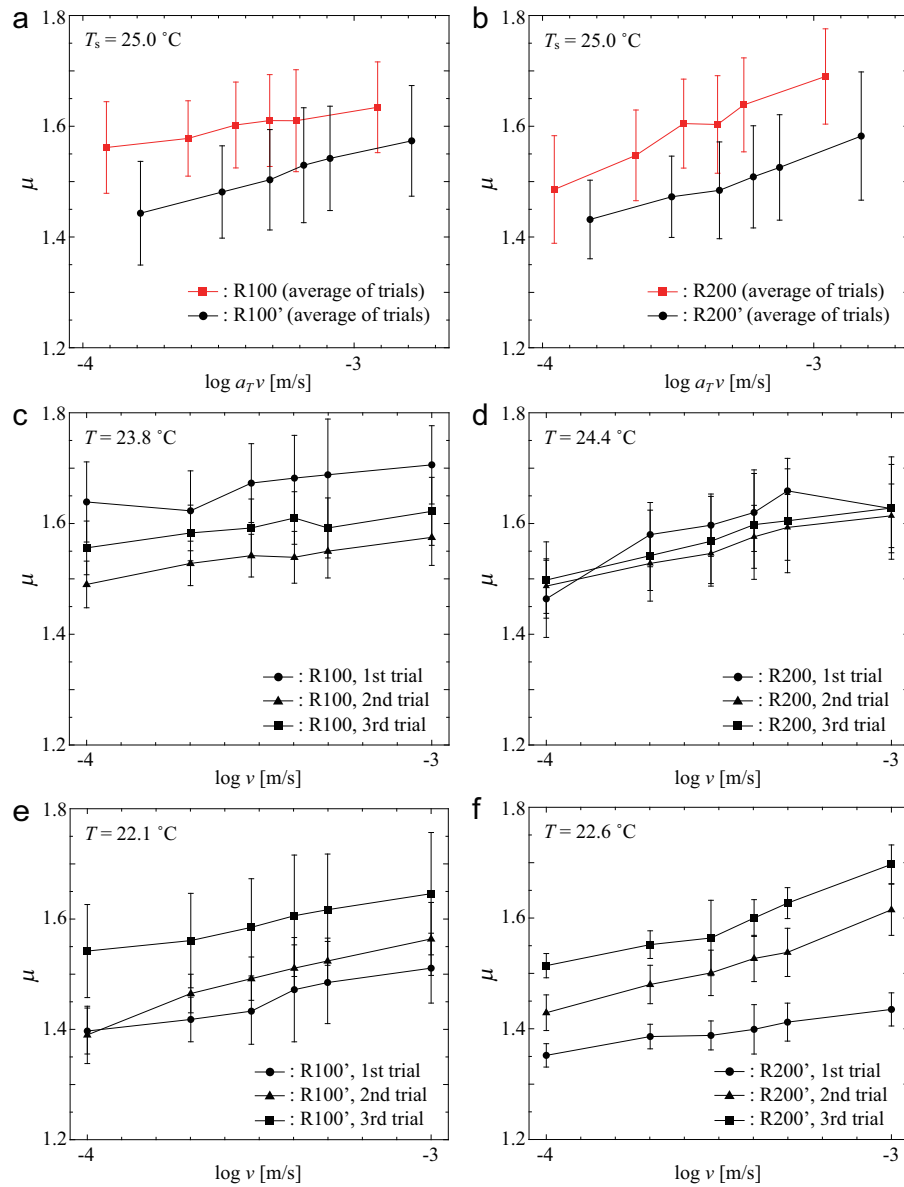


Fig. 8 (a) Average friction coefficients of R100 and R100' at a reference temperature of 25 °C; (b) average friction coefficients of R200 and R200' at a reference temperature of 25 °C; friction coefficients of (c) R100, (d) R200, (e) R100' and (f) R200' for each trial.

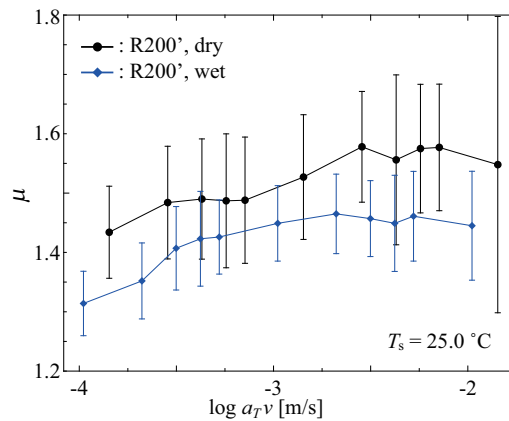


Fig. 9 Average friction coefficients of R200' under dry and wet conditions where the reference temperature is 25 °C, and the sliding velocity ranges from 0.1 to 10 mm/s.

between the friction forces on the dry and wet surfaces is equal to the adhesive shear force. As mentioned previously, flash temperature reduces the friction coefficient at higher sliding speed. It is true that the rate of increase in μ reduced with increasing v and then stick-slips occurred when $\log v > -2.5$, which indicates a bifurcation from steady sliding to stick-slip instability (Baumberger et al., 1994, Heslot et al., 1994, Ozaki and Hashiguchi, 2010, Ozaki et al., 2013).

4.2. Numerical assessment

In Section 4.1, the experiment revealed that the increments of $\mu(v)$ did not vary greatly among all measurements in the range of $10^{-4} \leq v \leq 10^{-3}$ mm/s, and the existence of adhesion friction was shown by the friction test held under a wet condition. We here conduct sensitivity analyses of pure rubber hysteresis based on Persson's theory and compare the results with the experimental results in terms of the magnitude and increment of $\mu(v)$.

With the surface description given in Table 2 and the viscoelastic model shown in Fig. 6, we estimate the friction coefficient from Eqs. (7) and (8) without nonlinear viscoelastic behavior; i.e., $\beta = 1$. In Fig. 10(a), the solid lines show the calculated μ_h as a function of the logarithm (base 10) of v for the different magnifications $\zeta_{\max}$, which corresponds to the upper cut-off wavenumber according to $q_1 = \zeta_{\max} q_0$, and the dot-dashed curve shows the measured friction coefficient for R200', which we here indicate as μ_{exp} . When $\zeta_{\max} = 3000$, the amplitude of μ_h agrees well with that of μ_{exp} at the lower side of sliding speed. However, the difference increases as v increases. Conversely, the rate of increase in μ_h comes close to that of μ_{exp} with decreasing $\zeta_{\max}$, and they best agree at $\zeta_{\max} = 250$, although the value of μ_h becomes smaller across the whole speed range. This means that the scale around $\zeta_{\max} = 250$ is the valid magnification for the prediction of the hysteresis friction. The obtained $\zeta_{\max}$ corresponds to $q_1 \approx 1.27 \times 10^6 \text{ m}^{-1}$ ($\lambda \approx 4.96 \text{ }\mu\text{m}$), the order of which agrees well with that of other experiment estimations (Westermann et al., 2004, Lorenz et al., 2011). In the case that $v = 10^{-4} \sim 10^{-2.7}$ m/s and $\zeta_{\max} = 250$, the operative frequency for the calculation of μ_h , which corresponds to $[q_0, q_1]$, ranges from 0 to 403.4 Hz.

Figure 10(b) shows A/A_0 , which is calculated using Eq. (3), as a function of ζ with varying sliding velocity. It is clear that the ratio of the real contact area is of the order of $10^{-4} \sim 10^{-5}$ for magnifications from 250 to 3000 and that there are only a few differences in A/A_0 among the measuring velocities for the fixed magnification. Considering $\varepsilon_0 = \sigma_0/E'(0) = 2450/(26.722 \times 10^6) \approx 9.17 \times 10^{-5}$, the macroscopic normal strain of sliding rubber is infinitesimal. However, the normal stress for the large magnification remarkably increases along with a sudden decrease in A/A_0 . Therefore, the normal strain should not be considered small at such a microscopic scale.

We next conduct sensitivity analysis of μ_h for a change in β , accounting for the effect of nonlinear viscoelastic behavior. Figures 11(a)–(c) show $\mu_h(v)$ –curves for the three representative parameters $\beta = 0.8, 0.6$ and 0.4 , respectively. The results in Figs. 11(a)–(c) reveal that all the hysteresis friction coefficients increase at each magnification with decreasing β , which is attributed to the increasing real contact area described by Eq. (14). For example, the rate of increase can be simply calculated as $A \rightarrow 0.25A$ when $\zeta_{\max} = 250$ and $\beta = 0.8$, because $\tan \delta$ is at most 0.2 in the operative frequency domain ranging from 0 to 403.4 Hz (see Fig. 6) and $\tan^2 \delta / \beta^2$ can be ignored in the integral range. Figure 11(d) focuses on the increment of the friction coefficient at $\zeta_{\max} = 250$; $\Delta\mu_h$ changes from 0.085 to 0.182 as β decreases from 1 to 0.4. The

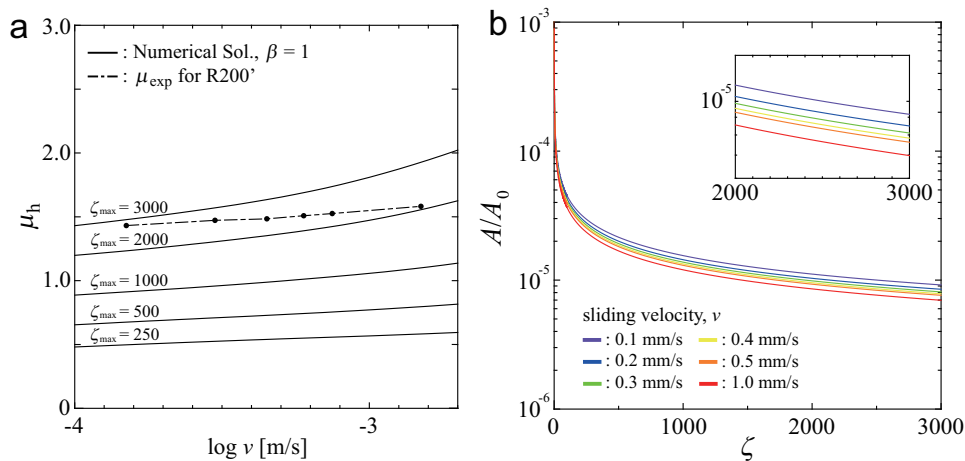


Fig. 10 (a) Calculated μ_h curves as a function of v for five different magnifications (solid lines) and the experimental curve of R200' (dot-dashed curve); (b) the calculated relative area of real contact A/A_0 as a function of ζ at the different sliding velocities.

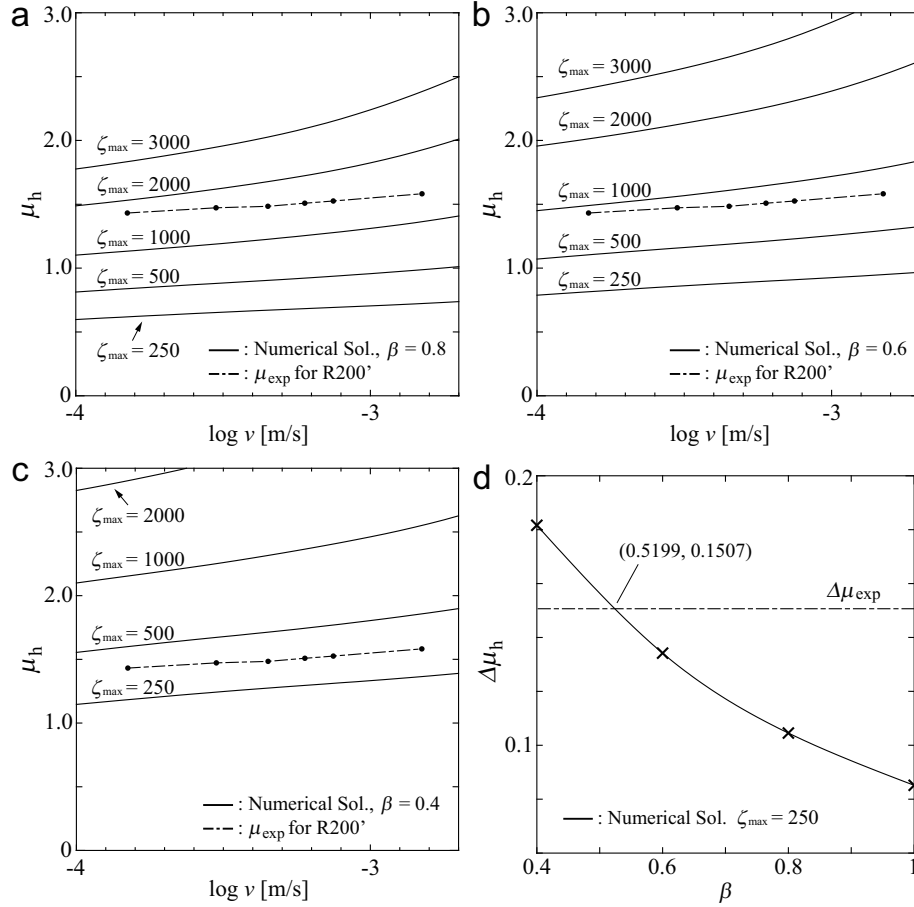


Fig. 11 Calculated $\mu_h(v)$ curves accounting for nonlinear viscoelastic behavior of the filled rubber: (a) $\beta = 0.8$; (b) $\beta = 0.6$; (c) $\beta = 0.4$, and (d) the increment of the hysteresis friction coefficient, $\Delta\mu_h = \mu_h(v_{\max}) - \mu_h(v_{\min})$, simulated for a change in β to compare with measurements.

plot shows that $\Delta\mu_h$ at $\beta = 0.5199$ is consistent with the measurement $\Delta\mu_{\text{exp}} = 0.1507$, accounting only for the hysteresis contribution of friction.

5. Discussion

The previous section verified that the nonlinear viscoelastic behavior of filled rubber is an important factor determining μ_h because a reduction in β increases A/A_0 and μ_h . It is necessary for further detailed prediction to measure the effective viscoelastic modulus for large strain and to estimate the effective strain the rubber undergoes during sliding. An effective procedure was studied by Lorenz et al. (2013a). In our experiment, the nominal strain is $\varepsilon_0 \approx 9.17 \times 10^{-5}$. Therefore, the complex elastic modulus measured at strain amplitude of 0.1 % is adequate on a macroscale. However, the ratio of the real contact area is less than of order 10^{-4} and effective strain at the microscale, $\varepsilon = A_0 \varepsilon_0 / A$, is estimated to be of order unity. Strain sweep testing in general shows that the dynamic storage modulus of carbon-black-filled rubber decreases less than half under such large strain (Payne, 1964, Payne, 1965, Barrés et al, 2003), which is acceptable for our numerical assessment that the simulated rubber coefficient $\beta < 1/2$ is in good agreement with μ_{exp} . The complete prediction of μ_h requires the true complex elastic modulus as a function of the strain amplitude changing across the multiple sizes of asperities.

We next consider the additional factor pertaining to the adhesive effect, which is well known to be important to rubber friction on a dry surface (Gal et al., 2005, Gal and Kluppel, 2008, Lorenz et al., 2011, Lorenz et al., 2013a). The adhesive force at the interface directly contributes to the shear force of friction; i.e., the total friction coefficient is given by

$$\mu = \mu_h + \mu_a, \quad \mu_a = \frac{\tau_s}{\sigma_0} \frac{A}{A_0}, \quad (15)$$

where μ_h is the hysteresis friction coefficient derived using Eq. (7) and μ_a is the adhesive friction coefficient due to the shear stress τ_s on the real contact area A . The true interfacial shear stress is attributed to microscopic bond breaking (i.e.,

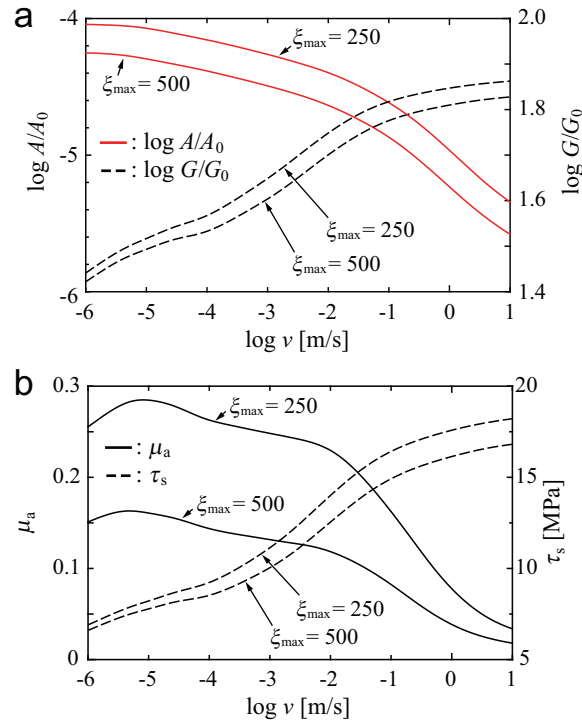


Fig. 12 Elements required to predict the adhesive friction coefficient μ_a : (a) A/A_0 and G/G_0 ; (b) τ_s and μ_a . Each function is calculated in the range of crack propagating velocity from 10^{-6} to 10 m/s when $\zeta_{\max} = 250$ with $\beta = 0.4$ or $\zeta_{\max} = 500$ with $\beta = 0.5$. Other details of the analysis are described in Appendix A.

the origin of wear), and the predictive modeling of τ_s has been developed in terms of surface free energy (Gal et al., 2005, Gal and Kluppel, 2008) or crack propagating energy (Lorenz et al., 2011, Persson and Brener, 2005b, Persson et al., 2005c). We here adopt the latter adhesion model, and the concrete calculation of τ_s is summarized in Appendix A.

Figure 12(a) shows the ratio of crack propagating energy G/G_0 in the sliding velocity range from 10^{-6} to 10 m/s, estimated using Eqs. (A.1)–(A.4), when $\zeta_{\max} = 250$ with $\beta = 0.4$ or $\zeta_{\max} = 500$ with $\beta = 0.5$. These conditions of $\zeta_{\max}$ and β will be later adopted to compare the calculated μ in Eq. (15) with the measured μ under dry and wet conditions. Note that, in the calculation of μ_a , we assume that the crack propagating velocity is equal to the sliding velocity of the rubber friction. In Fig. 12(a), we include A/A_0 calculated in Eq. (3) under the same conditions of $\zeta_{\max}$ and β . Subsequently, we analyze τ_s and μ_a using Eqs. (A.5) and (15) as shown in Fig. 12(b).

The results in Figs. 12(a) and (b) reveal the following. The order of the calculated τ_s is similar to that obtained in an earlier experiment (Lorenz et al., 2011). Nevertheless, μ_a predicted in this study is 6 ~ 10 times smaller than the measured friction coefficient (see Fig. 8) since the ratio of real contact area A/A_0 is small owing to the low load stress. Alternatively, A/A_0 rapidly drops with increasing v in contrast to the increase in G/G_0 , which explains why μ_a asymptotically approaches zero as v increases. The obtained frictional property is the conventional behavior that the viscoelastic peak of the friction coefficient due to adhesion occurs at a low sliding speed (Grosch, 1963, Moore, 1980). By comparing the cases for $\zeta_{\max} = 250$ with $\beta = 0.4$ and $\zeta_{\max} = 500$ with $\beta = 0.5$, it is found that all the four elements, A/A_0 , G/G_0 , τ_s and μ_a , are reduced with increasing the magnification $\zeta_{\max}$ throughout the range of the sliding velocity.

Figure 13 shows the calculated friction coefficient including the hysteresis and adhesion terms as $\mu = \mu_h + \mu_a$ for $\zeta_{\max} = 250$ and $\beta = 0.4$, and the result is that the magnitude of μ agrees well with that of μ_{exp} for R200' as shown in Fig. 8(b). To investigate whether the predicted adhesion friction is well estimated, Figures 14(a) and (b) compare the calculated hysteresis and adhesion friction with the experiment value under the dry and wet conditions in Fig. 9. In Fig. 14(a), the predicted μ (bold solid line) for $\zeta_{\max} = 250$ and $\beta = 0.4$ is also in good agreement with μ_{exp} on the dry substrate at the lower sliding speed. However, the calculated μ_h (bold dashed line) is lower than μ_{exp} on the wet substrate at the lower sliding speed, thus, μ_a is overestimated. On the other hand, in the case of $\zeta_{\max} = 500$ and $\beta = 0.5$, the difference of μ_{exp} between dry and wet conditions is well represented by μ_a (see Fig. 14(b)). However, the rate of increase in the coefficient with respect to sliding velocity enlarges due to the double scale of $\zeta_{\max}$, thus, μ_h becomes overestimated with increasing v .

There are two possible explanations for this inconsistency. One explanation is that a temperature-induced vibration

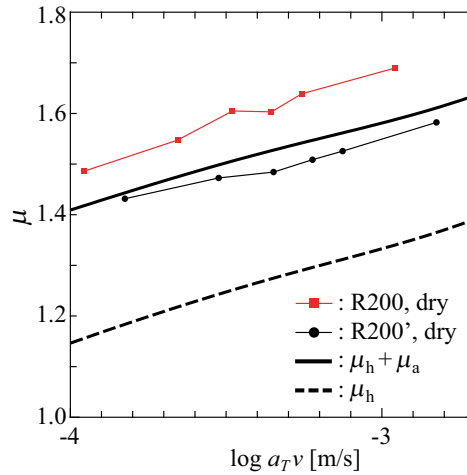


Fig. 13 Correlations of the numerical hysteresis and adhesion friction and the measurements as shown in Fig. 8(b); the bold dashed line indicates the friction coefficient calculated with only the hysteresis term under the conditions of $\zeta_{\max} = 250$ and $\beta = 0.4$, and the bold solid line indicates both the hysteresis and adhesion terms under the same conditions.

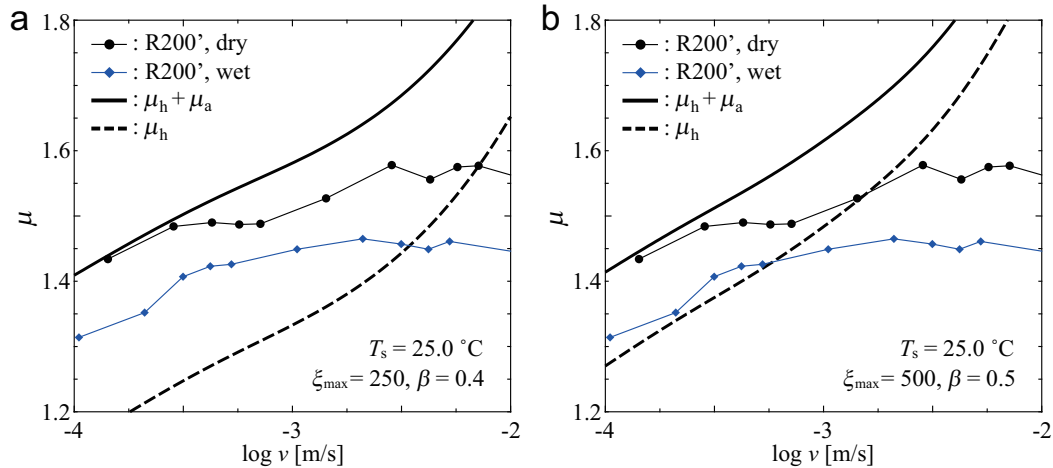


Fig. 14 Correlations of the numerical hysteresis and adhesion friction and the measurements under dry and wet conditions as shown in Fig. 9: the bold dashed and solid lines indicate the friction coefficients calculated by μ_h and $\mu = \mu_h + \mu_a$, respectively, under the conditions of (a) $\zeta_{\max} = 250$ with $\beta = 0.4$ or (b) $\zeta_{\max} = 500$ with $\beta = 0.5$.

arises at the local interface in the range of high sliding velocities. Since the rubber is softened by flash temperature, partial local slips, which are referred to as *precursors*, easily occur (Ozaki et al., 2014). The propagation of these precursor events along the sliding direction generates macroscopic stick-slip motion and reduces the friction coefficient (Rubinstein et al., 2004, Ozaki et al., 2014). During sticking-slipping, the rubber friction is no longer dominated by pure hysteresis deformation, and the predictions thus disagree with the measurements at a higher sliding velocity. The other explanation is that the concise form of Eq. (14) with a scaling parameter β is not adequate to precisely represent the nonlinear viscoelastic behavior of the filled rubber. Different from our model, which uniformly scales the storage elastic modulus, the complex elastic modulus of filled elastomers indeed exhibits complicated amplitude- and frequency-dependent properties (Höfer and Lion, 2009), where the hysteresis loss is strongly related to properties of the molecular chain microstructure of filled rubber such as the volume fraction and the distribution morphology of carbon black (Tomita et al., 2008). The constitutive approach of nonlinear viscoelasticity has been extensively developed to represent such an observed material behavior (Höfer and Lion, 2009, Österlöff et al., 2014) and the frictional formulation of Persson's theory should be enhanced to extend the viscoelastic modulus with these appropriate models.

6. Summary and conclusion

We studied a prediction model for the friction coefficient of a rubber block sliding over a rough and rigid surface, employing Persson's theory, and compared the results obtained with experimental measurements. First, we briefly ex-

plained hysteresis rubber friction theory developed by Persson for rubber sliding over a nonadhesive surface with rigid asperities of multiple scales, which was formulated with rubber viscoelasticity and the power spectrum of surface roughness. Subsequently, we measured the self-affine fractal property of the surface with laser microscopes and identified the complex elastic modulus of the rubber sample employing a 20-term generalized Maxwell model, fitting the master curves obtained in a vibration test. With surface and rubber specimens, we conducted friction tests at low sliding velocities with light squeezing, and qualitatively assessed the experimental results using the prediction model described as the addition of the hysteresis and adhesive friction coefficients: $\mu = \mu_h + \mu_a$.

The main insights obtained can be summarized as follows. Employing Persson's theory, we estimated the wavenumber range of the surface height as $[q_0, q_1] \approx [5.07 \times 10^3, 1.27 \times 10^6]$ (1/m), where the hysteresis of rubber viscoelastic deformation was purely integrated. The obtained upper cut-off wavenumber corresponded to the smallest wavelength of about 5 μm , the order of which was similar to that of estimations in earlier studies (Westermann et al., 2004, Lorenz et al., 2011). Accounting for nonlinear viscoelastic behavior, we modeled the corrective complex elastic modulus with a scaling parameter β , and found that the proposed model modestly agreed with both the magnitude of the measured friction coefficient and the rate of increase in the coefficient with respect to sliding velocity when $\zeta_{\max} = q_1/q_0 = 250$ and $\beta = 0.4 \sim 0.5$. In addition, we simulated the adhesion force derived from the crack propagating energy (Lorenz et al., 2011). We then showed that the adhesion friction for $\zeta_{\max} = 250$ was overestimated to compare with the experiment measurements under dry and wet conditions and that our model described using a scaling parameter β needs to be improved for the precise prediction of rubber friction including the hysteresis and adhesion effects.

In this study, we adjusted the two parameters q_1 and β to coincide with the measured friction coefficient. We thus revealed the qualitative nature of friction for filled rubber on a rough surface, such as the change in both the friction coefficient and the real contact area along with sliding velocity and nonlinear viscoelastic behavior of rubber. As it now stands, it is hard to quantitatively distinguish these parameters because a single parameter of a friction coefficient is consolidated from multiple physical elements containing the two parameters. To that end, we need to conduct further experiments at several temperatures to investigate the rate of the friction coefficient in a wider range of sliding velocities. We also need to extend the experiments using several combinations of types of compound rubber and surfaces, including unfilled rubber and silicon-filled rubber, and lubricated surfaces, and to establish an assessment framework from the resulting database.

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Appendix A. Estimation of shear stress due to crack propagation by Persson's theory

Employing theory on energy dissipation at an opening crack between rubber and a substrate (Lorenz et al., 2011, Persson and Brener, 2005b, Persson et al., 2005c), we here estimate the adhesive shear stress and the related friction coefficient due to crack propagation of the interfacial rubber. In Ref. (Lorenz et al., 2011), the relationship between the crack propagation energy $G(v)$ and the crack-tip radius $a(v)$ with crack propagating velocity v satisfies the similarity rule

$$\frac{a(v)}{a_0} = \frac{G(v)}{G_0}, \quad (\text{A.1})$$

where G_0 and a_0 relate to a very slowly propagating crack. With numerical consideration, the left-hand side of Eq. (A.1) is described as (Lorenz et al., 2011)

$$\frac{a_0}{a} = 1 - \left(1 - \frac{E(\omega_0)}{E(\omega_1)}\right) \frac{\int_0^{\xi^*} d\xi F(\omega_0 e^{\xi}) \text{Im}[1/E(\omega_0 e^{\xi})]}{\int_0^{\xi^*} d\xi \text{Im}[1/E(\omega_0 e^{\xi})]}, \quad (\text{A.2})$$

where

$$F(\omega_0 e^{\xi}) = \left[1 - e^{2(\xi - \xi^*)}\right]^{1/2}, \quad (\text{A.3})$$

and

$$\xi^* = \ln \left(\frac{2\pi\nu a_T}{a\omega_0} \right), \quad (\text{A.4})$$

where a_T is the shift factor in Eq. (13). Furthermore, $E(\omega_0) \approx E(0)$ and $E(\omega_1) \approx E(\infty)$. In our calculation, we set sufficiently small and large values as $\omega_0 = 1.0 \times 10^{-6}$ rad/s in the rubbery region and $\omega_1 = 1.0 \times 10^9$ rad/s in the glassy region, respectively.

The shear stress τ_f , resulting from the crack propagation of the interfacial rubber, can be described as

$$\tau_s = \frac{G(v)}{G_0} \frac{G_0}{\ell}, \quad (\text{A.5})$$

where ℓ is the size of the real contact region. In the calculation of τ_s , $G(v)/G_0$ was obtained from Eqs. (A.1)–(A.4). We here set $a_0 = 1$ nm and $G_0/\ell = 0.25$ MPa in reference to (Lorenz et al., 2011).

Equations (A.1)–(A.5) provide the numerical predictions of $\log G/G_0$ and τ_s corresponding to our experiment data (see Figs. 12(a) and (b)).

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