

Evaluation of dynamic absorber to suppress subharmonic nonlinear vibration in car powertrain

Sofian ROSBI*, Takahiro RYU**, Takashi NAKAE**, Kenichiro MATSUZAKI***, Atsuo SUEOKA****, Yoshihiro TAKIKAWA***** and Yoichi OOI*****

* School of Mechatronic Engineering, Universiti Malaysia Perlis,
Pauh Putra Campus, 02600 Arau, Perlis, Malaysia
E-mail: sofian.rosbi@gmail.com

** Department of Mechanical and Energy Systems Engineering, Faculty of Engineering, Oita University,
700 Dannoharu, Oita-shi, Oita, 870-1192, Japan

*** Graduate School of Science and Engineering, Kagoshima University,
Korimoto 1-21-24, Kagoshima-shi, Kagoshima, 890-8580, Japan

**** Kyushu Polytechnic College,
1665-1 Shii, Kokura Minami-ku, Kitakyushu-shi, Fukuoka, 802-0985, Japan

***** Core Component Engineering Department, Engineering Division, Aisin AW Co., Ltd.,
10 Takane, Fujii-cho, Anjo-shi, Aichi, 444-1192, Japan

Received 18 December 2014

Abstract

This paper analyzed the suppression method of the nonlinear vibration called subharmonic vibration of order $1/2$ in the car powertrain by using a dynamic absorber. In the car powertrain, the torsional forced vibration caused by engine explosions leads to inhibition of the ride quality. Thus, in the torque converter of an automatic transmission car, a piecewise-linear spring called damper is used to reduce the transmission of the forced vibration. However, the subharmonic vibration occurred in the actual vehicle, when the spring restoring characteristics is around the switching point. The fundamental vibration frequency of the subharmonic vibration is half to the engine forced vibration frequency. Although the design of the dynamic absorber to suppress the forced vibration has been established for the linear systems, the optimum design to suppress the subharmonic vibration of order $1/2$ has not yet been investigated. The powertrain is modeled by multi degree-of-freedom system as an actual vehicle, including engine, torque converter, transmission gears and wheels. Equation of motion is developed with also considering spring restoring force of piecewise-linear spring. The numerical result shows subharmonic vibration occurs when the engine excitation frequency is almost twice of the second mode of natural frequency, and the result shows a good agreement with the experimental data. Then, the dynamic absorber is introduced to suppress the occurrence of subharmonic vibration theoretically. In this paper, the optimum design of the dynamic absorber to suppress the subharmonic vibration is discussed. The numerical result shows the optimally designed dynamic absorber in the natural frequency and the damping ratio is capable of suppressing the occurrence of the subharmonic vibration.

Key words : Forced vibration, Vibration of rotating body, Nonlinear vibration, Dynamic absorber, Powertrain, Piecewise-linear spring, Subharmonic vibration of order $1/2$

1. Introduction

In recent years, automatic transmission has become widely used, and the number of cars that equipped with automatic transmission is increasing. Automatic transmission cars are easy for driving operation compared to the manual transmission car. The disadvantage of the automatic transmission cars is lower fuel efficiency compared to the manual transmission cars. In order to overcome this disadvantage, a lock-up clutch system in the torque converter is developed. The lock-up clutch system is composed of a piston with a damper and a frictional material. The damper is in

the form of the springs that support the piston.

In the latest trends in engine technologies, diesel engines and higher power engines are widely used. These engines contribute to the strong torsional vibration in powertrain systems. On the other hand, lower number cylinders of engines are developed to realize high efficiency of fuel consumption, and lower limit of the torsional vibration frequency due to engine explosions is decreased. This setting also contributes to the increment of the vibration level in powertrain systems.

To address this problem, the damper of the lock-up clutch needs to be designed to absorb the torsional vibration effectively. The damper with low stiffness contributes to reduce the fluctuation in rotational speed caused by the explosions in the engine combustion chamber, but it is difficult to use the low stiffness springs which is applicable to the wide range of the static torque for the lack of the space.

As the solution, the damper is designed using a piecewise-linear spring with three stiffness stages. This piecewise-linear spring has the characteristics that the spring constant increases with the increase of the static torque. This type of spring can realize wide range of the restoring torque characteristics in a small space.

However, a nonlinear vibration, called subharmonic vibration of order 1/2, occurred because of the nonlinearity by the piecewise-linear spring. In the subharmonic vibration of order 1/2, the main frequency component is half of the forced vibration frequency. The subharmonic vibration of order 1/2 occurred around the switching point in the piecewise-linear restoring torque characteristics. There have been some studies about the modeling of powertrain (Crowther and Zhang, 2004), the nonlinear vibrations in the mechanical systems (Kim, et al., 2005), the subharmonic vibrations (Al-shyyab and Kahraman, 2003), and the piecewise-linear spring properties (Kawamura, et al., 2006). However, the occurrence mechanism and the countermeasure for the subharmonic vibration of order 1/2 in the actual torque converter with the piecewise-linear spring have not yet been studied.

In the present study, we focused on the countermeasure to suppress the subharmonic vibration of order 1/2 by using a dynamic absorber. For the liner systems, the optimum designs of the dynamic absorber for the single and multi degree-of-freedom systems had already been established (Den Hartog, 1985) (Seto et al., 1984). Since the fundamental vibration frequency is half of the engine excitation frequency, these design methods are not applicable to the subharmonic vibration. Then, we performed modeling and numerical analysis to investigate the optimum design of the dynamic absorber to suppress the occurrence of the subharmonic vibration.

2. Outline of subharmonic vibration of order 1/2

The automatic transmission consists of a torque converter and a gear train. The torque converter is located between the engine and the gear train to transmit the torque induced by the engine to the gear train. The inside of the torque converter is filled with automatic transmission fluid (ATF). Figure 1 shows a schematic diagram of a torque converter. The torque converter consists of a pump impeller, a turbine runner and a stator. As the torque converter transmits torque through the fluid, the rotational speed of the turbine runner is slower than that of the pump impeller, which causes the inefficiency associated with torque converter. In order to overcome this disadvantage, a lock-up clutch, which connects the input and output sides through friction, is used.

The piston is supported by springs called damper. The function of damper is to reduce the fluctuation in rotating speed caused by the explosions in the engine combustion chamber.

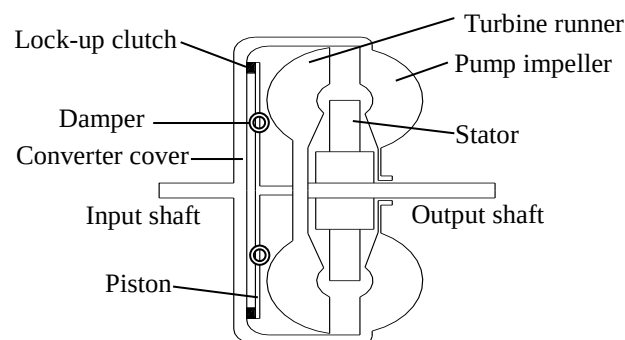


Fig. 1 Schematic diagram of torque converter.

A traveling test using an actual vehicle was conducted in order to investigate the occurrence mechanism of the subharmonic vibration of order 1/2 (henceforth "subharmonic vibration"). Figure 2 shows the Campbell diagram at the turbine runner. The abscissa shows the engine speed and the ordinate shows the vibration frequency. The color scale represents the vibration amplitude of the rotational speed of the turbine runner. The engine frequency in Fig. 2 represents the component of the torsional vibration frequency due to the engine explosions. The engine frequency increases in association with the increase of the engine speed. There is a strong vibration inside the dashed line. This vibration is the subharmonic vibration.

The characteristics of the subharmonic vibration are summarized as follows.

- (1) The vibration frequency of the subharmonic vibration is half of the engine excitation frequency. Figure 3 shows waveform at the pump impeller (upper figure) and the turbine runner (lower figure) when the subharmonic vibration occurred. The vibration period of the pump impeller and the turbine runner is twice of that of the engine torque fluctuation. According to the frequency analysis of the engine torque, it is confirmed that the 1/2 order component to the main excitation frequency is very small.
- (2) Subharmonic vibrations occur in all state of gear when the lock-up clutch is locked.
- (3) The damper is designed as a piecewise-linear spring with three stages. Figure 4 shows the spring restoring torque characteristics of the damper. The abscissa ϕ represents the angular displacement of the damper, and the ordinate $T(\phi)$ represents the restoring torque. The subharmonic vibration occurred when static torque is near the switching point between the second and third stages of the spring restoring torque (Point B in Fig. 4). However, the subharmonic vibration did not occur near the switching point between the first and second stages of the spring restoring torque (Point A in Fig. 4).
- (4) The engine frequency range when the subharmonic vibration occurred is almost twice of the second mode's natural frequency of the system.

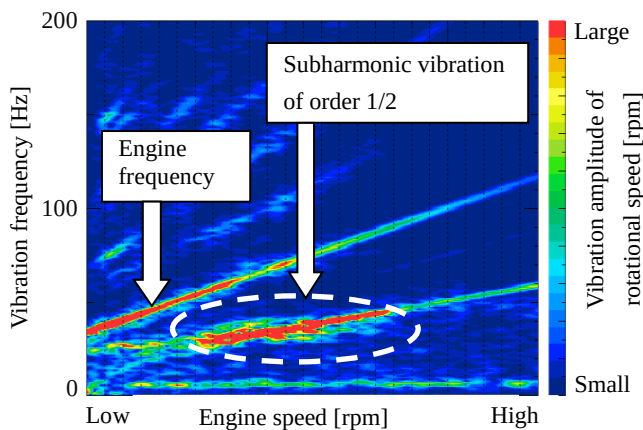


Fig. 2 Campbell diagram at turbine runner.

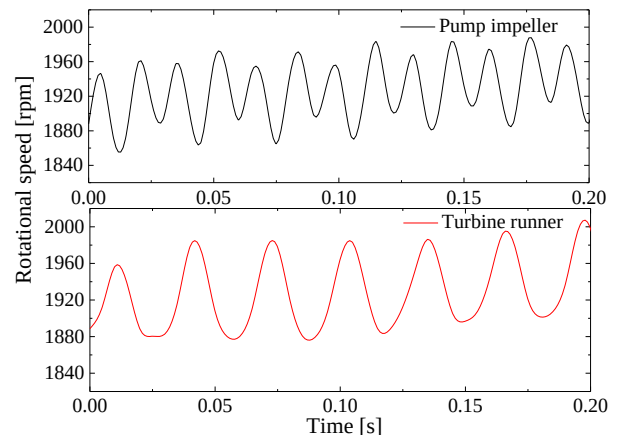


Fig. 3 Waveform of subharmonic vibration.

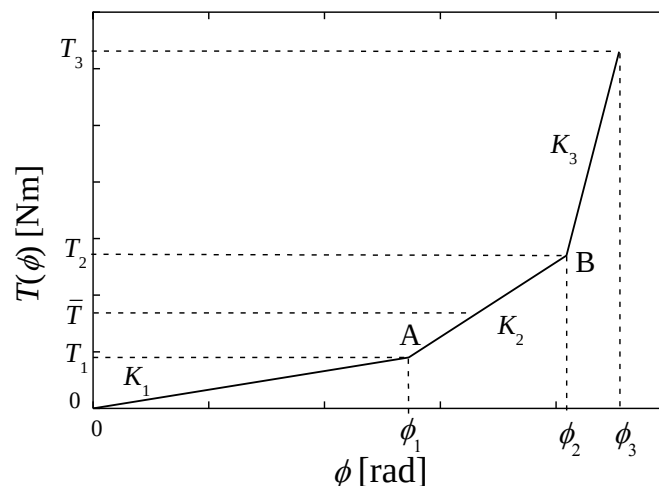


Fig. 4 Piecewise-linear spring restoring torque characteristics.

3. Theoretical analysis

In order to analyze the generation of the subharmonic vibration, a system based on the actual automatic transmission is modeled. In the experiment, the subharmonic vibration occurred in the powertrain for FF vehicle. In this paper, only the fifth gear structure is considered in the modeling.

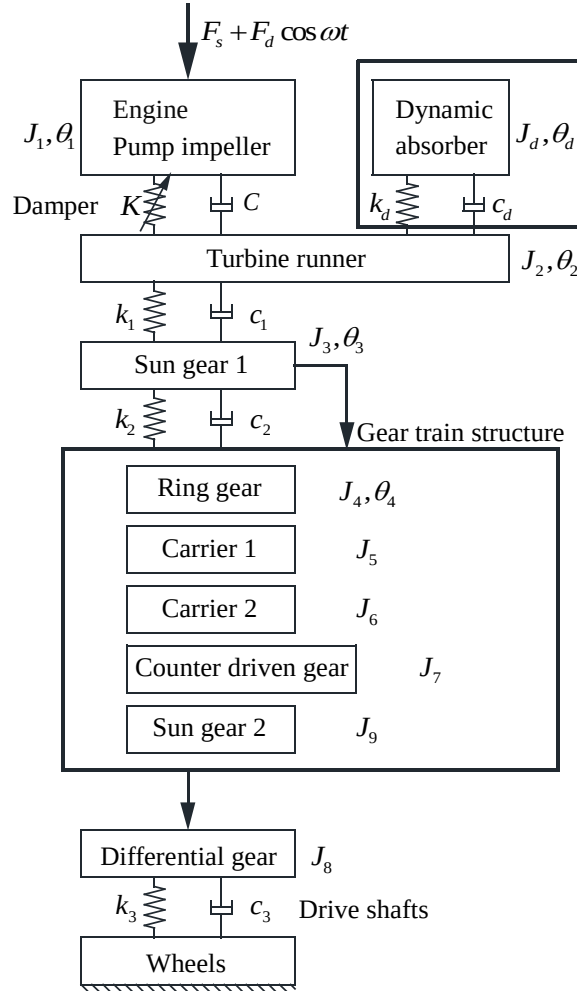


Fig. 5 Analytical model for fifth gear.

Figure 5 shows the modeling of the gears train in fifth gear. This model includes the torque converter, the gear train and the differential gear. The gear train is composed by the Ring gear, Carrier 1, Carrier 2, Counter driven gear and Sun gear 2. The damper, the shafts in the gear train and the drive shafts are modeled by rotational springs. The wheels are assumed to be the fixed end. The system is modeled by 4 degree-of-freedom system. In the figure, $J_1, \dots, J_9$ are the moments of inertia of the components, k_1, k_2 , and k_3 are the rotational spring constants, $\theta_1, \theta_2, \theta_3$, and θ_4 are the angular displacements for each element, c_1, c_2 , and c_3 is damping coefficients for each damping element, F_s is the static engine torque, F_d is the dynamic engine torque, and ω is the angular frequency of the dynamic torque. The inertia of the engine is included in the inertia of the pump impeller J_1 . The damper is modeled by the nonlinear rotational spring K of the piecewise-linear spring, and the dashpot with the linear damping coefficient C . As shown in Fig. 4, the spring constants of the first, second and third stages in the piecewise-linear spring are K_1, K_2 , and K_3 respectively, and the angular displacement and the torque at Points A and B are (ϕ_1, T_1) and (ϕ_2, T_2) , respectively.

The angular displacements of the Carrier 1, Carrier 2, Counter driven gear, Sun gear 2 and the differential gear are represented by $n_i\theta_3 + m_i\theta_4$ ($i=1, \dots, 5$), respectively, where n_i and m_i are the constants which relate to the gear ratios between the components. In this gear train, $m_1=0$ and $m_5=m_2$.

The equation of motion for a 4-degree-of-freedom system is written as:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{n}(\mathbf{x}) + \mathbf{F}, \quad \mathbf{x} = [\theta_1 \quad \theta_2 \quad \theta_3 \quad \theta_4]^T \quad (1)$$

where the right superscript T indicates transposed representation. In Eq. (1), $\mathbf{x}$ is angular displacement vector, $\mathbf{n}(\mathbf{x})$ is the spring restoring torque vector of the damper, $\mathbf{F}$ is the external torque vector, $\mathbf{M}$ is mass matrix, $\mathbf{C}$ is damping matrix and $\mathbf{K}$ is stiffness matrix.

The mass matrix $\mathbf{M}$ is represented by Eq. (2).

$$\mathbf{M} = \begin{bmatrix} M_{11} & 0 & 0 & 0 \\ 0 & M_{22} & 0 & 0 \\ 0 & 0 & M_{33} & M_{34} \\ 0 & 0 & M_{43} & M_{44} \end{bmatrix} \quad \left. \begin{aligned} M_{11} &= J_1, \quad M_{22} = J_2, \quad M_{33} = J_3 + n_1^2 J_5 + n_2^2 J_6 + n_3^2 J_7 + n_4^2 J_8 + n_5^2 J_9, \\ M_{44} &= J_4 + m_2^2 J_6 + m_3^2 J_7 + m_4^2 J_8 + m_5^2 J_9, \\ M_{34} &= M_{43} = n_2 m_2 J_6 + n_3 m_3 J_7 + n_4 m_4 J_8 + n_5 m_5 J_9 \end{aligned} \right\} \quad (2)$$

In Eq. (2), the components M_{11} , M_{22} , M_{33} , M_{34} , M_{43} and M_{44} are the function of n_i , m_i and the inertia of each parts $J_1, \dots, J_9$.

The damping matrix $\mathbf{C}$, the stiffness matrix $\mathbf{K}$ and the spring restoring torque vector $\mathbf{n}(\mathbf{x})$ are represented in Eqs. (3) and (4).

$$\mathbf{C} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 \\ 0 & C_{32} & C_{33} & C_{34} \\ 0 & 0 & C_{43} & C_{44} \end{bmatrix} \quad \left. \begin{aligned} C_{11} &= C, \quad C_{12} = C_{21} = -C, \quad C_{22} = C + c_1, \\ C_{23} &= C_{32} = -c_1, \quad C_{33} = c_1 + c_2 + n_4^2 c_3, \\ C_{34} &= C_{43} = -c_2 + n_4 m_4 c_3, \quad C_{44} = c_2 + m_4^2 c_3 \end{aligned} \right\} \quad (3)$$

$$\mathbf{K} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & K_{22} & K_{23} & 0 \\ 0 & K_{32} & K_{33} & K_{34} \\ 0 & 0 & K_{43} & K_{44} \end{bmatrix}, \quad \mathbf{n}(\mathbf{x}) = \begin{bmatrix} -T(\theta_1 - \theta_2) \\ T(\theta_1 - \theta_2) \\ 0 \\ 0 \end{bmatrix} \quad \left. \begin{aligned} K_{22} &= k_1, \quad K_{23} = K_{32} = -k_1, \\ K_{33} &= k_1 + k_2 + n_4^2 k_3, \quad K_{34} = K_{43} = -k_2 + n_4 m_4 k_3, \\ K_{44} &= k_2 + m_4^2 k_3 \end{aligned} \right\} \quad (4)$$

The external force vector $\mathbf{F}$ is described in Eq. (5).

$$\mathbf{F} = [F_s + F_d \cos \omega t \quad 0 \quad 0 \quad 0]^T \quad (5)$$

Equations from (1) to (5) are used to calculate nonlinear vibration in powertrain system. Shooting method (Aprille and Trick, 1972) is applied to solve the nonlinear equation of Eq. (1). The stability of the solution is determined by the eigenvalue analysis of the transition matrix which is given by the variational equation of Eq. (1) (Aprille and Trick, 1972). In the process of the numerical integration method in the Shooting method, the switching time of the piecewise-linear spring between the time steps is calculated precisely by using the Newton-Raphson method.

In order to calculate the natural frequencies and the natural modes of the powertrain system, linear equations for each stages of the damper stiffness are introduced. The amendments of the Eq. (1) and Eq. (4) are shown in Eq. (6).

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \tilde{\mathbf{K}}\mathbf{x} = \mathbf{0} \quad \left. \begin{aligned} \tilde{\mathbf{K}} &= \mathbf{K} + \begin{bmatrix} K & -K & 0 & 0 \\ -K & K & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad K = K_1, K_2, K_3 \end{aligned} \right\} \quad (6)$$

In the present paper, the use of a dynamic absorber is proposed in order to suppress the subharmonic vibration. As shown in Fig. 5, a dynamic absorber with a moment of inertia J_d , a damping coefficient c_d and a spring coefficient k_d is attached to the turbine runner. The amended equation of motion for the turbine runner, based on Eq. (1), and the equation of motion for the dynamic absorber are shown in Eq. (7).

$$\left. \begin{aligned} J_2 \ddot{\theta}_2 + c_1 (\dot{\theta}_2 - \dot{\theta}_3) + k_1 (\theta_2 - \theta_3) + C (\dot{\theta}_2 - \dot{\theta}_1) + c_d (\dot{\theta}_2 - \dot{\theta}_d) + k_d (\theta_2 - \theta_d) &= T (\theta_1 - \theta_2) \\ J_d \ddot{\theta}_d + c_d (\dot{\theta}_d - \dot{\theta}_2) + k_d (\theta_d - \theta_2) &= 0 \end{aligned} \right\} \quad (7)$$

4. Results of numerical calculations and discussions

4.1 Natural frequencies and natural modes

Table 1 shows the natural frequencies when the spring constant of the damper K in Eq. (6) is set to K_1 , K_2 and K_3 , respectively. Here, the dynamic absorber is not attached. As the subharmonic vibration is related to the second mode in the experiment, we are focusing on second mode. In Table 1, the natural frequencies of the second modes for K_1 , K_2 and K_3 are 24.5, 26.97 and 39.80 Hz respectively. Figure 6 (a), (b) and (c) show up to the 4th natural mode for K_1 , K_2 and K_3 respectively. In the second modes for K_1 , K_2 and K_3 , θ_2 , θ_3 , and θ_4 vibrate out of phase with respect to θ_1 .

Table 1 Natural frequencies [Hz].

Spring constant K	Modes			
	1 st	2 nd	3 rd	4 th
K_1	7.88	24.5	95.3	671
K_2	9.95	26.97	97.1	671
K_3	13.07	39.80	112	671

Modes	1 st	2 nd	3 rd	4 th
θ_1				
θ_2				
θ_3				
θ_4				

(a) $K = K_1$

Modes	1 st	2 nd	3 rd	4 th
θ_1				
θ_2				
θ_3				
θ_4				

(b) $K = K_2$

Modes	1 st	2 nd	3 rd	4 th
θ_1				
θ_2				
θ_3				
θ_4				

(c) $K = K_3$

Fig. 6 Natural modes for each spring constant.

4.2 Forced vibration analysis for subharmonic vibration

To clarify the characteristics of the subharmonic vibration, the numerical calculations for the system without dynamic absorber are performed. In this nonlinear forced vibration analysis, we focused on the turbine runner amplitude (θ_2) to compare with the experimental results in Figs. 2 and 3. As to the standard parameters, the dynamic torque F_d is set to 100 Nm and the damping coefficient of the damper C is set to 1 Nms/rad.

Figures 7 (a) and (b) show the frequency response curves at the turbine runner when the static torques are $F_s = \bar{T}$ and T_2 in Fig. 4. The abscissa represents the excitation frequency due to the engine explosion f , and the ordinate represents the peak to peak amplitude of the angular displacement. The black and red lines show the stable and unstable solutions respectively. The static torque $\bar{T}$ is set far from the switching points of Points A and B and T_2 is set to switching point of Point B. The frequency response curve of Fig. 7 (a) is the same result of the linear forced vibration analysis using $K = K_2$. The resonance frequency around $f = 30$ Hz in Fig. 7 (a) is the main resonance of the second mode. In the case of Fig. 7 (b), since this vibration straddle two stages of spring constants K_2 and K_3 , the main resonance frequency is between the natural frequencies of 26.97 and 39.80 Hz. In Fig. 7 (b), the symbols $\circ$, $\triangle$, and $\square$ represent the saddle-node, hopf and flip (period-doubling) bifurcations respectively. In the saddle-node bifurcation, the jump phenomenon occurs. In the hopf bifurcation, the quasi-periodic vibration occurs. In the flip bifurcation, the subharmonic vibration of order 1/2 occurs.

The large amplitude area around $f = 60$ Hz in Fig. 7 (b) is the subharmonic vibration of order 1/2. In the case of $F_s = \bar{T}$, there is no resonance around this area. Even though there are no natural frequencies around this area in Table 1, the subharmonic resonance occurs because of the nonlinearity of the restoring torque characteristics. The maximum amplitude of the subharmonic vibration is 0.029 rad. In addition, it is confirmed analytically that the subharmonic vibration occurs not only on Point B in Fig. 4, but also around near Point B.

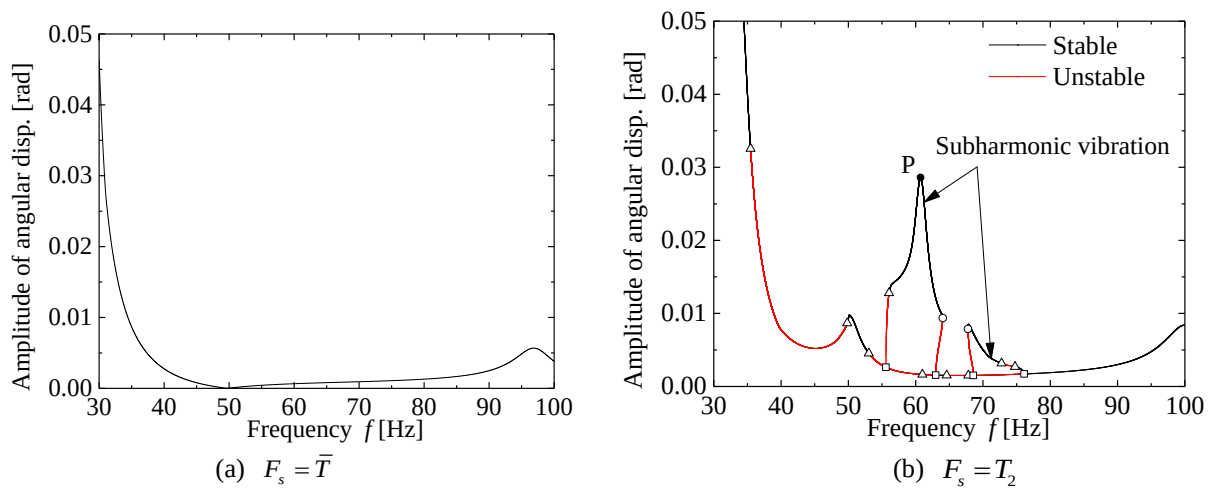


Fig. 7 Frequency response curve at turbine runner.

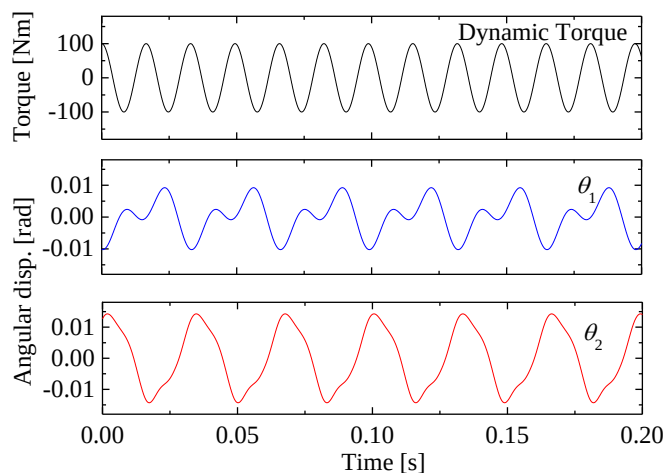


Fig. 8 Vibration waveform of subharmonic vibration at Point P ($f = 60.7$ Hz).

Figure 8 shows the waveforms of the subharmonic vibration at Point P in Fig. 7 (b). The excitation frequency by engine is $f = 60.7$ Hz. The top, middle and bottom figures represent the waveforms of input dynamic torque, the pump impeller (θ_1) and the turbine runner (θ_2) respectively. It is found that the period of θ_2 is twice of the input dynamic torque and that the waveforms of θ_1 and θ_2 are similar to the experimental results in Fig. 3.

Figure 9 shows the frequency analysis of θ_2 at Point P in Fig. 7 (b). It is found that the fundamental frequency of θ_2 with 30.4 Hz which is the half of the excitation frequency by engine. This is the characteristics of the subharmonic vibration of order 1/2.

The vibration mode of the subharmonic vibration at Point P in Fig. 7 (b) is examined. The magnitude and phase corresponding to the component of fundamental frequency with 30.4 Hz are shown in Table 2. It is found that the vibration mode is similar to the 2nd natural modes shown in Fig. 6.

There is a peak of amplitude around $f = 50$ Hz in Fig. 7 (b). This is the superharmonic resonance of order 2 concerned with the 3rd mode, but the peak amplitude is small. This phenomenon is also caused by the nonlinearity of the piecewise-linear spring. In the result in Fig. 7 (a), there is no such resonance.

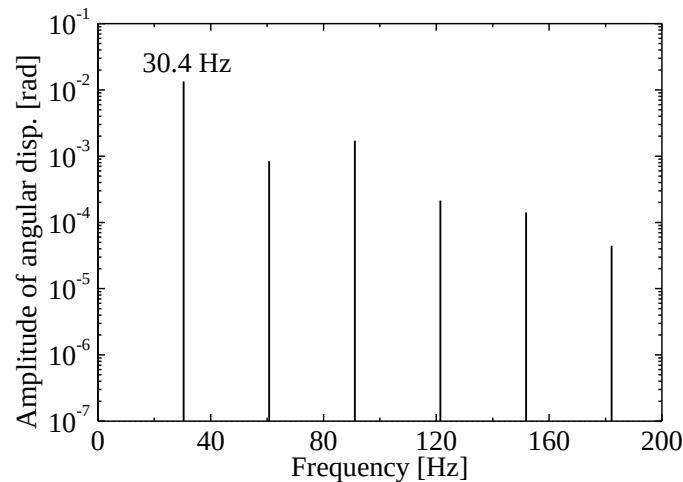


Fig. 9 Frequency analysis of turbine runner ($f = 60.7$ Hz).

Table 2 Vibration mode of subharmonic vibration.

	Magnitude	Phase [deg.]
θ_1	1	0
θ_2	2.25	-178.92
θ_3	2.47	-178.99
θ_4	2.65	-179.04

4.3 Effect of dynamic absorber on subharmonic vibration

The effectiveness of a dynamic absorber attached to the turbine runner in suppressing the subharmonic vibration was next investigated. The damping ratio ζ_d and the natural frequency f_d of the dynamic absorber are defined as follows:

$$\zeta_d = \frac{c_d}{2\sqrt{J_d k_d}}, \quad f_d = \frac{1}{2\pi} \sqrt{\frac{k_d}{J_d}} \quad (8)$$

Here we focus on the suppression of the subharmonic vibration at Point P in Fig. 7 (b). The frequency ratio between the natural frequency of the dynamic absorber f_d and the excitation frequency at Point P $f_e (= 60.7$ Hz) is defined as Eq. (9).

$$\eta = f_d / f_e \quad (9)$$

Since the fundamental vibration frequency of the subharmonic vibration is half of the excitation frequency due to the engine explosion, half of the excitation frequency at Point P is defined as $f'_e (= f_e / 2 = 30.4\text{Hz})$. The frequency ratio between the natural frequency of the dynamic absorber f_d to f'_e is also defined as Eq.(10).

$$\eta' = f_d / f'_e \quad (10)$$

According to the optimum design for the linear systems, the natural frequency of the dynamic absorber is set close to the excitation frequency. Firstly, the effect of dynamic absorber whose natural frequency is close to the excitation frequency is evaluated. The inertia of the dynamic absorber J_d is set to $0.3J_2$. The damping ratio ζ_d is set to 0.1. Figure 10 shows the frequency response curve at the turbine runner when the frequency ratio η is equal to 0.9. The abscissa represents the excitation frequency due to the engine explosion f , and the ordinate represents the peak to peak amplitude of the angular displacement at the turbine runner. The maximum amplitude for the subharmonic vibration is 0.030 rad which is almost the same value as the amplitude at Point P. It is concluded that dynamic absorber does not give any suppressive effect when the natural frequency of the dynamic absorber is set close to the engine forced frequency f_e . Even though the result when $\eta=1.0$ is omitted, it was confirmed that the dynamic absorber with $\eta=1.0$ did not give any suppressive effect. The result shows that the optimum design to suppress the subharmonic vibration by using the dynamic absorber is completely different from that designed for the linear systems.

Then, the suppressive effect when the natural frequency of the dynamic absorber is set to about half of the excitation frequency is discussed. Figures 11 (a), (b) and (c) show the frequency response curves at the turbine runner when the frequency ratio η' is set to 0.8, 0.9 and 1.0 respectively. The abscissa represents the excitation frequency due to the engine explosion f , and the ordinate represents the peak to peak amplitude of the angular displacement. The maximum amplitude of the subharmonic vibration for the frequency ratio $\eta'=0.8, 0.9$ and 1.0 are 0.018 rad, 0.007 rad and 0.018 rad respectively. Compared with the result of Fig. 7 (b), the maximum amplitudes of the subharmonic vibration are reduced. Especially for the case of $\eta'=0.9$, the main branch around $f=60\text{Hz}$ is completely suppressed. Figure 11 concluded that the subharmonic vibration can be suppressed by attaching the dynamic absorber whose natural frequency is close to half of the engine excitation frequency, and there exists an optimum value of the frequency ratio η' to suppress the subharmonic vibration.

Next, the effect of the damping ratio of the dynamic absorber on the subharmonic vibration is evaluated. The natural frequency ratio η' is set to 0.9. Figures 12 (a) and (b) show the frequency response curves at the turbine runner when the damping ratio ζ_d is set to 0.05 and 0.2 respectively. The abscissa represents the excitation frequency due to the engine explosion f , and the ordinate represents the peak to peak amplitude of the angular displacement. The maximum amplitudes of the subharmonic vibration for the damping ratio $\zeta_d=0.05$, and 0.2 are 0.0141 and 0.0138 rad respectively. In Fig. 12, the main branches of the subharmonic vibration around $f=60\text{Hz}$ still exist. It is concluded that there also exists the optimum value of the damping ratio of the dynamic absorber ζ_d to suppress the subharmonic

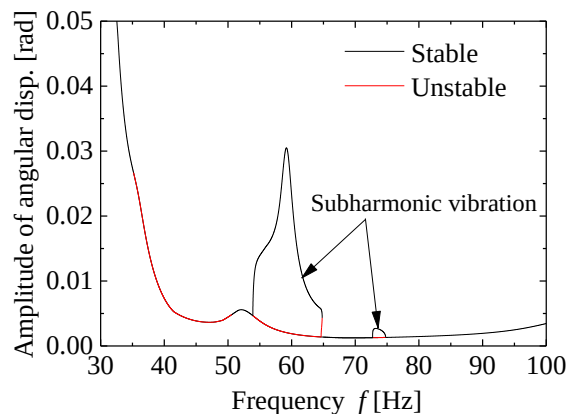


Fig. 10 Frequency response curve when dynamic absorber is attached ($\eta = 0.9$).

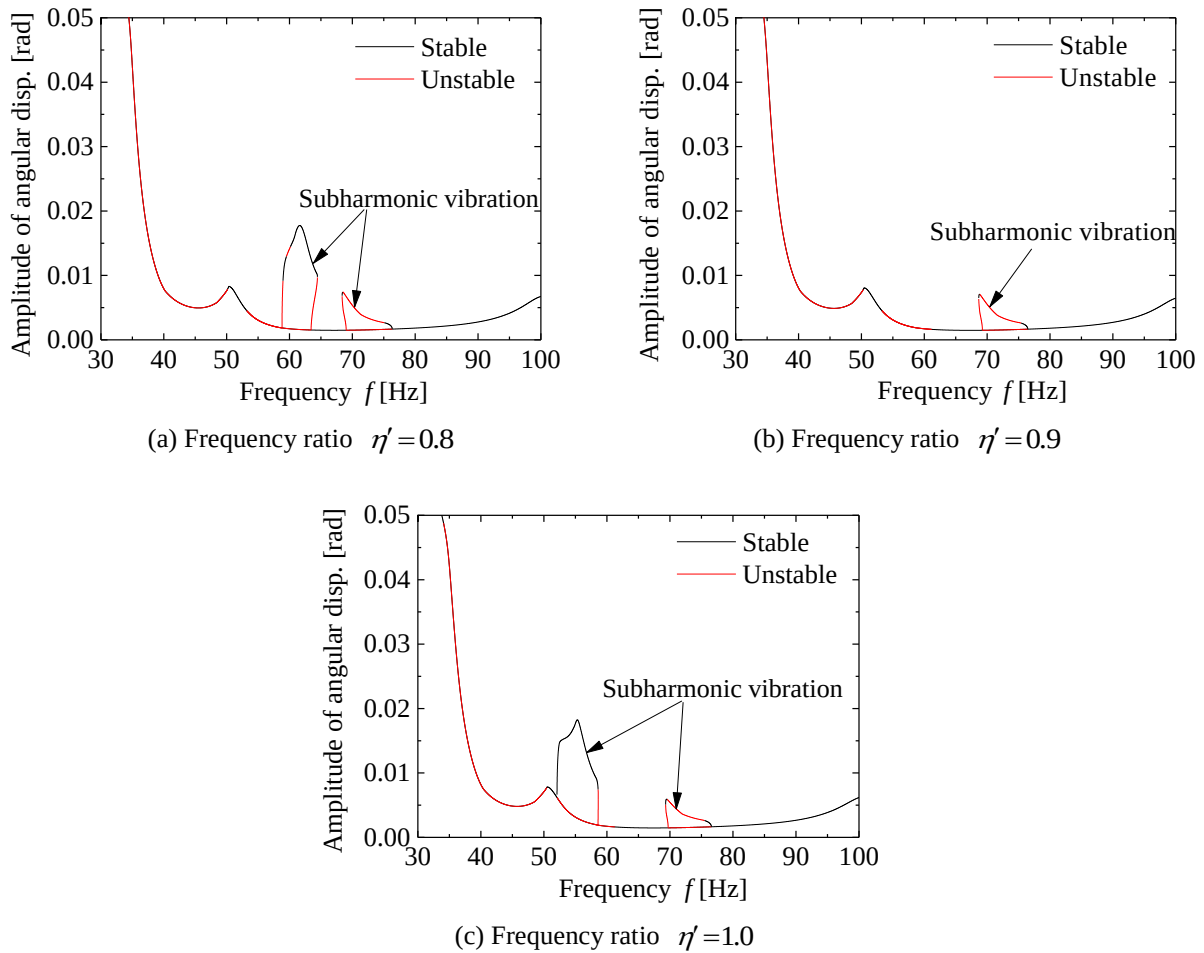


Fig. 11 Frequency response curve when dynamic absorber is attached.

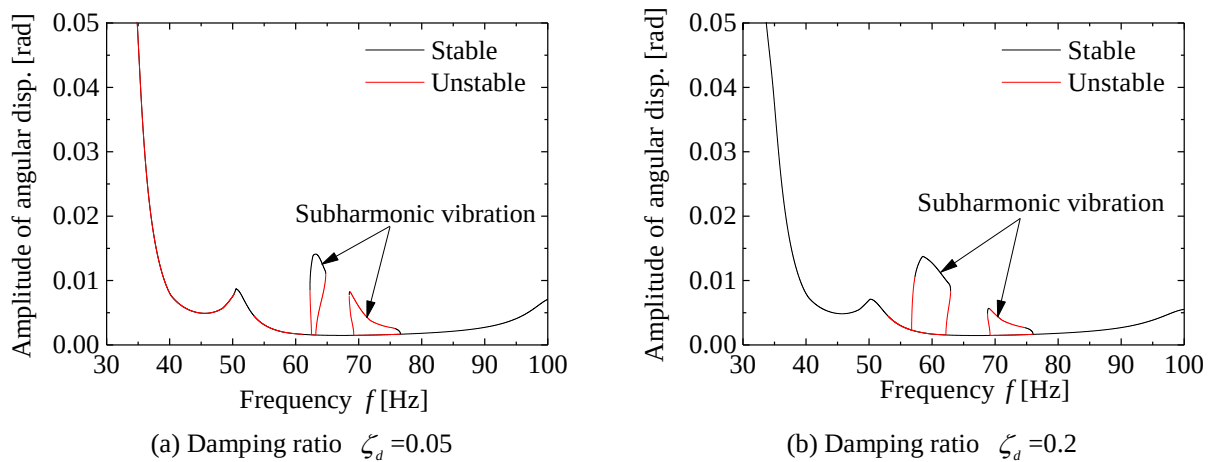


Fig. 12 Effect of damping ratio of dynamic absorber on subharmonic vibration ($\eta' = 0.9$).

vibration.

Figure 13 shows the effect of inertia of the dynamic absorber on the subharmonic vibration. The damping ratio ζ_d is set to 0.1. The abscissa represents the frequency ratio η' and the ordinate represents the maximum peak to peak amplitude of the angular displacement for the subharmonic vibration at the turbine runner. The inertias of the dynamic absorber J_d are changed to $0.1J_2$, $0.2J_2$, $0.3J_2$ and $0.4J_2$. From Fig. 13, it can be seen that the values of η' are about 0.9, and larger inertia contributes to the suppression of the subharmonic vibration.

Figure 14 shows the maximum peak to peak amplitude of the subharmonic vibration at the turbine runner when the damping ratio ζ_d and the frequency ratio η' of the dynamic absorber are changed. The inertia of dynamic absorber

J_d is set to $0.3J_2$. From Fig. 14, it can be seen that there exist the optimum values for the damping ratio ζ_d and the frequency ratio η' to suppress the subharmonic vibration. In the range of this calculation using the dynamic absorber with $J_d=0.3J_2$, the dynamic absorber with $\zeta_d=0.1$ and $\eta'=0.9$ is the most effective to suppress the subharmonic vibration.

The optimum design of the dynamic based on the modal analysis is going to be investigated in the future.

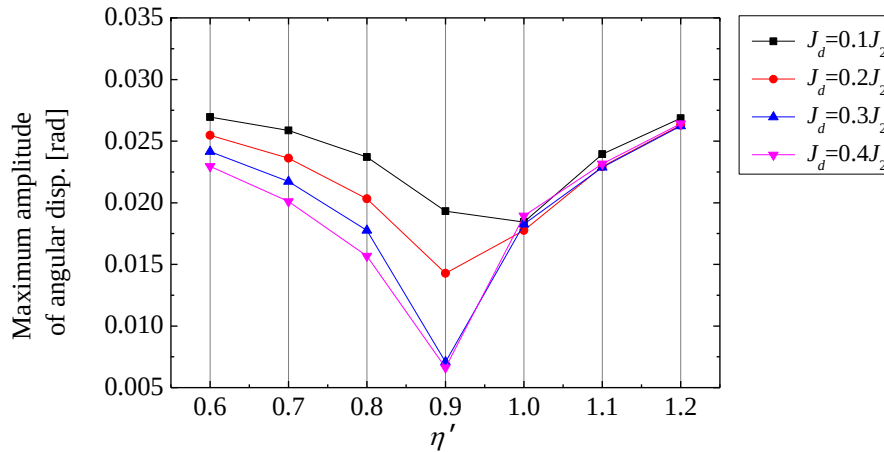


Fig. 13 Effect of inertia of dynamic absorber ($\zeta_d=0.1$).

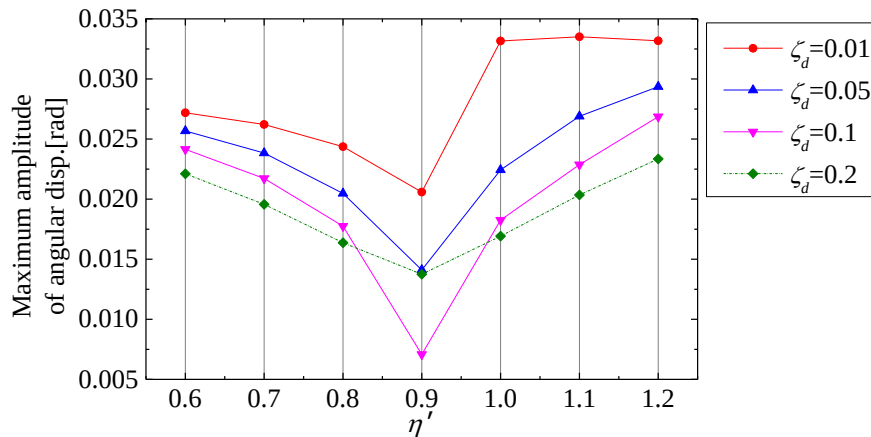


Fig. 14 Optimum design of dynamic absorber ($J_d = 0.3J_2$).

5. Conclusions

This paper focuses on the optimum design of the dynamic absorber to suppress the subharmonic vibration of order 1/2 in an automatic transmission powertrain theoretically. This paper concludes:

- (1) The dynamic absorber does not give any suppressive effect when the natural frequency of the dynamic absorber is set close to the engine forced frequency.
- (2) The subharmonic vibration of order 1/2 can be suppressed by attaching the dynamic absorber whose natural frequency is close to half of the engine excitation frequency.
- (3) The larger inertia of the dynamic absorber contributes to the suppression of the subharmonic vibration.
- (4) There exist the optimum values of the dynamic absorber damping ratio and the frequency ratio of the dynamic absorber natural frequency to the excitation frequency to suppress the subharmonic vibration. When the optimum dynamic absorber is attached to the system, the subharmonic vibration is suppressed.

References

- Al-shyyab, A. and Kahraman, A., Non-linear dynamic analysis of a multi-mesh gear train using multi-term harmonic balance method : sub-harmonic motions, *Journal of Sound and Vibration*, Vol.279 (2005), pp.417–451.
- Aprille, T. and Trick, T., A computer algorithm to determine steady-state response of nonlinear oscillators, *Transactions on Circuit Theory*, Vol.19 (1972), pp.354–360.
- Crowther, A.R. and Zhang N., Torsional finite elements and nonlinear numerical modelling in vehicle powertrain dynamics, *Journal of Sound and Vibration*, Vol.284 (2005), pp.825–849.
- Den Hartog, J.P. , *Mechanical Vibrations* (1985), Dover, New York, p.87.
- Kawamura, S., Suzuki, M., Zahid, H. and Minamoto, H., A modeling concept of a mechanical system having a piecewise linear spring property for its diagnosis, *Applied Acoustics*, Vol.70 (2009), pp.1393–1399.
- Kim, T.C., Rook, T.E. and Singh, R., Super- and sub-harmonic response calculations for a torsional system with clearance nonlinearity using the harmonic balance method, *Journal of Sound and Vibration*, Vol.281 (2005), pp.965–993.
- Seto, K., Iwanami, K. and Takita, Y., *Vibration Control of Multi-Degree-of-Freedom Systems by Dynamic Absorbers*, *Transactions of the Japan Society of Mechanical Engineers (Series C)*, Vol.50, No.485 (1984), pp.1962–1969 (in Japanese).