

Levitation of a flexible rotor supported by hybrid magnetic bearings

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Abstract

Recently, it has become necessary for rotors of centrifugal compressors, industrial rolls, and other devices to be less weighted, have a long span, and operate at high rotation speed to increase productivity. Such rotors are more likely to have a high order mode than other types of rotors at the same rotational speed, and may become unstable. To solve this problem, active magnetic bearings are used to support the flexible rotor. However, because a general active magnetic bearing uses bias current to control the bearing force, it consumes a large amount of electricity. To reduce power consumption, a hybrid active magnetic bearing (HAMB) that consists of electromagnets and permanent magnets is used. The HAMB fabricated in this work has negative stiffness in both radial and angular directions. Two HAMBs, one at each end of a flexible rotor arranged horizontally, are installed to levitate it. A PID controller with a notch filter is used to control levitation. With only a PID controller, levitation of the flexible rotor becomes unstable because the flexible mode stimulates vibration. To settle the flexible mode vibrations, the notch filter is designed at the frequency of the flexible mode. Applying the notch filter to the displacement control loop has little effect on vibration decrement, whereas applying it against the angle control loop produces a remarkable effect. Therefore, the notch filter is applied only to the angle control loop. A rotation test is then implemented. The experimental results show that the HAMB-based control system stably passes through the second flexible mode rotation speed. Then, by using the advantage of the HAMB to reduce power consumption effectively, the ability of the system to reduce the control current to use zero power control is demonstrated, and, the results are compared with PID control when levitating the flexible rotor. Furthermore, a levitation test is conducted with linear quadratic regulator control.

Key words : High order mode, Hybrid active magnetic bearing, Flexible rotor, Levitation control, Notch filter

1. Introduction

A magnetic bearing utilizes magnetic force to levitate and support a rotor in the absence of physical contact. Hence, it has some advantages, including the lack of friction wear, which makes it maintenance free, improved rotation speed (Okada, et al., 1995), and superior vibration control due to the use of active control. Recently, industrial rotary machines such as centrifugal compressors, industrial rolls, and other devices require lightweight rotors with long spans and high rotation speed to improve productivity. However, a rotor that satisfies these requirements, has a high order bending mode and becomes unstable more easily than other rotors even if the rotation speed is in the same range. Similarly, on a “flexible rotor,” bending mode occurs within a range of operation speeds and causes the “spillover problem,” in which the system is unstable at high order modes which are not considered into the system model or omitted in the process of model reduction for controller design (Balas, 1978). Currently, a control method for solving this problem is being actively studied (Cui, et al., 1992).

Because a conventional active magnetic bearing must supply a large winding current to levitate the rotor at all times, it consumes a large amount of electricity. To solve this problem, a hybrid active magnetic bearing (HAMB) that

consists of electromagnets and permanent magnets is proposed (Komori, et al., 1996). The HAMB uses the permanent magnet to generate bias flux, which enables reduced electricity consumption. Although various forms of HAMB are suggested (Okada, et al., 2011), here we utilize a model in which an iron plate is set beside the stator electromagnet via the permanent magnet to pass the bias flux. This HAMB has negative spring stiffness not only in the radial direction but also in the angular direction. Two HAMBs are located in each side of the rotor and control the levitation. In our earlier system, levitation was unstable when an impulse disturbance was added to the rotor because the second flexible mode was stimulated and generated a resonance phenomenon. Therefore, a notch filter at second mode frequency is incorporated in this system to avoid stimulation of this mode. Additionally, the notch filter is only effective in the angular direction of the central PID controller. A rotation test carried out in this system indicated that it stably passes through the second flexible mode rotation speed. Moreover, to reduce electricity consumption relative to that required for PID control, the effect of using zero power control was clarified. Finally, the results of a levitation test carried out with a controller designed by the LQR control method are shown.

2. Experimental device and control system

The experimental set-up and its parameters are shown in Fig. 1 and Table 1, respectively. The flexible rotor is arranged horizontally and its shaft length is designed long so that the flexible mode occurs at a lower rotation speed for safety. Two HAMBs, one at each end, are installed on the flexible rotor so as to levitate it. One of them is shown in Fig. 2.

The structure of the HAMB and its levitation principle are shown in Fig. 3. A HAMB consists of a four-pole stator, an iron plate with a hole whose diameter is larger than that of the rotor, and permanent magnets placed between the two. Coils are wound on the pole of the stator, in addition, the upper and lower sides, as well as the left and right sides, are paired by windings so that the fluxes penetrate the rotor. Below, we define the four-pole stator as the control section and the iron plate as the bias section.

The bias fluxes generated by the permanent magnets pass through the bias section (Fig. 3, blue lines), and produce radial bias fluxes in the control section. On the other hand, by supplying current to the coils, control fluxes are produced in the control section (Fig. 3, red lines). Thus, an imbalance of magnetic flux density is generated in the air gaps between the rotor and stator, which in turn yields the bearing force. The bearing force is controlled by adjusting the currents. Because the control fluxes cannot flow through the bias section, the current does not affect the flux of the bias section. Hence, negative stiffness is produced only in the bias section.

The control system is shown in Fig. 4. Eddy current-type displacement sensors are placed beside the control section to reduce the non-collocation influence (Gosiewski and Kulesza, 2013) and detect the displacement of each rotor in the horizontal and vertical directions (Fig. 3). Because the sensor signals contain noise from the power amplifiers, low-pass filters (LPF) are used to decrease the influence of noise. The LPF is of the multiple feedback type, with a cut-off frequency of 5 kHz. The sensor signals are then transmitted to a digital signal processor (DSP) through analog-to-digital (A/D) converters, and coil currents are calculated by a control program in the DSP. Thereafter, the command values of the currents are sent to the power amplifiers through digital-to-analog (D/A) converters, whereupon currents are supplied to the coils. A switching amplifier is used for the power amplifier.

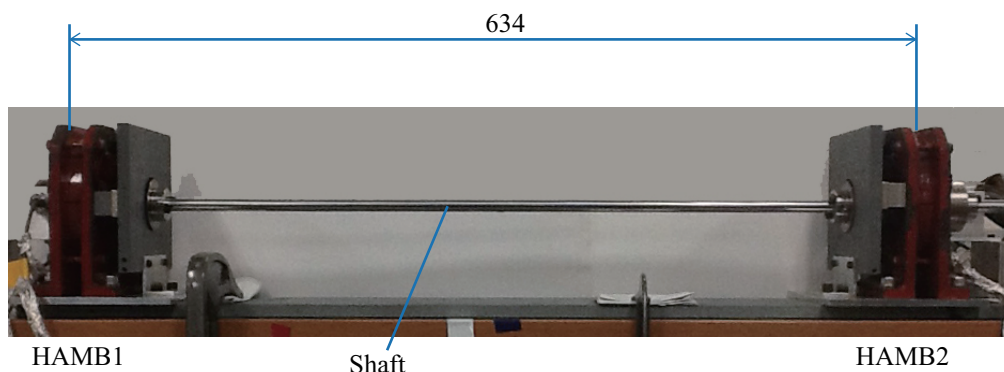


Fig. 1 Flexible rotor and bearing system.

Table 1 Parameters of Rotor-HAMB System

Shaft		HAMB Rotor	
Material	SUS304	Material	S45C
Radius	5 mm	Radius	20 mm
Length	800 mm	Length	90 mm
Mass	0.5 kg	Mass	0.9 kg
		Moment of inertia	$7 \times 10^{-4} \text{ kgm}^2$
		Polar moment of inertia	$1.8 \times 10^{-4} \text{ kgm}^2$

HAMB Stator		Permanent Magnet	
Pole area	$18 \times 20 \text{ mm}^2$	Type	NdFeB
Air gap	1 mm	Material	N-40
Coil turns	200	Size	$20 \times 12 \times 10 \text{ mm}^3$
Resistance	$3.1 \text{ } \Omega$	Number	3
Inductance	12.6 mH	Bias flux density	0.2 T

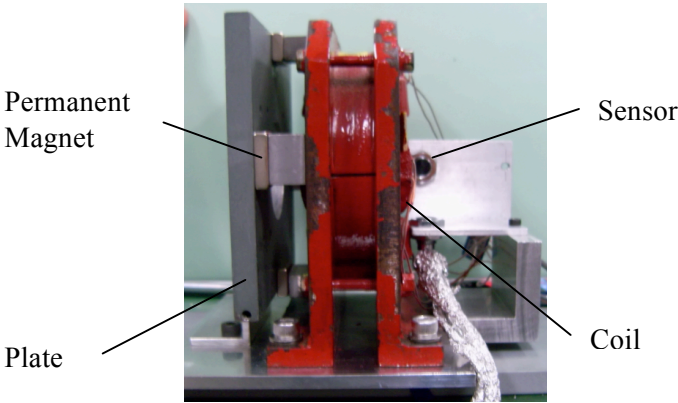


Fig. 2 Hybrid Active Magnetic Bearing.

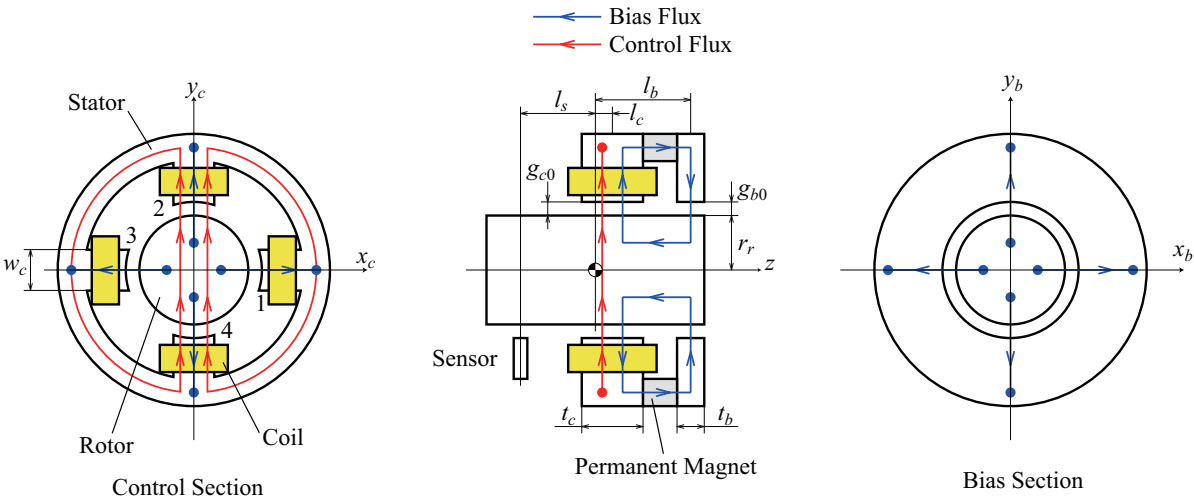


Fig.3 Structure of Hybrid Active Magnetic Bearing.

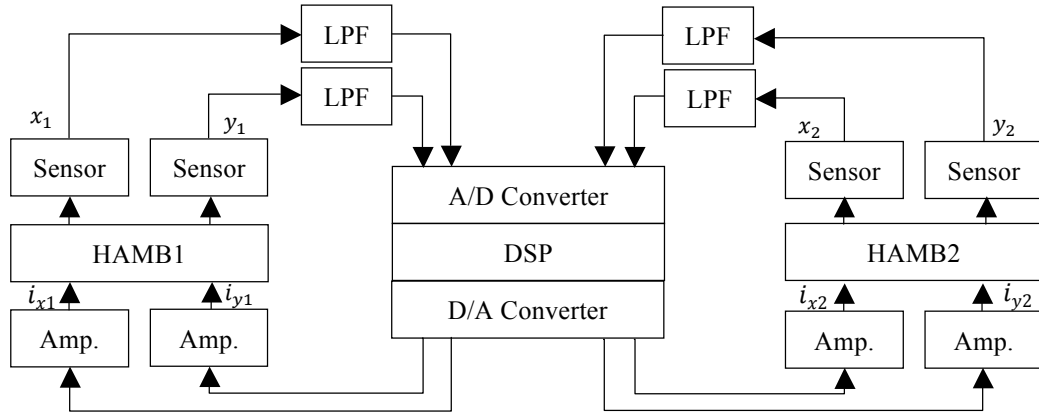


Fig. 4 Control System. Sensor signals are transmitted to a DSP through the low pass filters and A/D converters, and coil currents are calculated by a control program in DSP. Thereafter, the command values of the currents are sent to the power amplifiers through D/A converters, whereupon currents are supplied to coils.

3. Modeling of controlled object and controller

3.1 Modeling of magnetic bearing

Assuming that the flux density of the bias section is inversely proportional to the air gap, the magnetic flux density distribution B_b is expressed as

$$B_b(\theta) = \frac{B_{b0}}{g_{b0} - r_d \cos(\theta - \psi)} \quad (1)$$

where B_{b0} is the flux density at the equilibrium position, r_d and ψ are the radial displacement and direction of the rotor, respectively, and g_{b0} is the air gap of the bias section. The magnetic attractive force, f_{br} , produced by this magnetic flux density is approximated as

$$f_{br} = \int_0^{2\pi} \frac{B_b(\theta)^2}{2\mu_0} t_b r_r \cos(\theta - \psi) d\theta \approx \frac{B_{b0}^2}{\mu_0} \pi t_b r_r \frac{r_d / g_{b0}}{1 - r_d^2 / g_{b0}^2} \approx \frac{\pi t_b r_r B_{b0}^2}{\mu_0} \frac{r_d}{g_{b0}} \quad (2)$$

Then, the force in the x and y directions are expressed as follows:

$$f_{bx} = k_{bn} x_b \quad (3)$$

$$f_{by} = k_{bn} y_b \quad (4)$$

$$k_{bn} = \frac{\pi t_b r_r B_{b0}^2}{\mu_0 g_{b0}} = \frac{A_b B_{b0}^2}{2\mu_0 g_{b0}} \quad (5)$$

where x_b and y_b are the horizontal and vertical displacements, respectively, of the rotor at bias section, k_{bn} is the negative stiffness, μ_0 is the permeability of air, t_b is the axial length of the bias section, r_r is the radius of the rotor, and A_b is the area of the cross section of the bias section.

Next, the bearing forces are considered. The flux through the bias and control section is defined as Φ_b . Assuming no flux leakage, the following relation is obtained.

$$\Phi_b = B_{b0} A_b = 4B_{c0} A_c \quad (6)$$

where B_{c0} is the flux density of the control section at the equilibrium position, and A_c is the area of the cross section of the control section. Then,

$$B_{c0} = \frac{\Phi_b}{4A_c} = \frac{A_b}{4A_c} B_{b0} \quad (7)$$

Next, the flux densities of the four rotor gaps are considered. The flux densities generated by the permanent magnet are:

$$B_1 = \frac{B_{c0}}{1 - x_c / g_{c0}} \quad (8)$$

$$B_2 = \frac{B_{c0}}{1 - y_c / g_{c0}} \quad (9)$$

$$B_3 = \frac{B_{c0}}{1 + x_c / g_{c0}} \quad (10)$$

$$B_4 = \frac{B_{c0}}{1 + y_c / g_{c0}} \quad (11)$$

The fluxes generated by the coils are expressed as

$$B_x = \frac{\mu_0 N i_x}{g_{c0}} \quad (12)$$

$$B_y = \frac{\mu_0 N i_y}{g_{c0}} \quad (13)$$

where x_c and y_c are the rotor displacements on the control section, g_{c0} is the air gap of the control section, N is the winding number, and i_x and i_y are coil currents. The magnetic attraction forces produced by these magnetic flux densities are approximated as:

$$f_{cx} = k_{cn} x_c + k_i i_x \quad (14)$$

$$f_{cy} = k_{cn} y_c + k_i i_y \quad (15)$$

where

$$k_{cn} = \frac{2A_c B_{c0}^2}{\mu_0 g_{c0}} = \frac{A_b^2 B_{b0}^2}{8\mu_0 A_c g_{c0}} \quad (16)$$

$$k_i = \frac{2A_c B_{c0} N}{g_{c0}} = \frac{A_b B_{b0} N}{2g_{c0}} \quad (17)$$

Then, the force and moment that act on the center of gravity (COG) of the rotor are considered. The distances from the COG of the rotor to the control section, bias section, and sensor are defined as l_c , l_b and l_s , respectively. The direction from COG to the bias section is defined as the positive direction, hence, l_b and l_c are positive and l_s is negative in the case of Fig. 3. We define the displacements and the angles of the COG of the rotor as x_g , y_g , ψ_{xg} and ψ_{yg} . If the model of the shaft part is considered as rigid, the angle of the COG of the HAMB rotor as shown in Fig. 3 is equal to the angle of the shaft part. Then the angle can be calculated from two displacement sensors at both sides. We describe these below x direction only.

The displacements at the three sections are expressed by the displacement and the angle of the COG as follows:

$$x_c = x_g + l_c \psi_{xg} \quad (18)$$

$$x_b = x_g + l_b \psi_{xg} \quad (19)$$

$$x_s = x_g + l_s \psi_{xg} \quad (20)$$

Then, the radial force and moment are

$$\begin{bmatrix} f_{xg} \\ M_{xg} \end{bmatrix} = \begin{bmatrix} k_{cn} + k_{bn} & l_c k_{cn} + l_b k_{bn} \\ l_c k_{cn} + l_b k_{bn} & l_c^2 k_{cn} + l_b^2 k_{bn} \end{bmatrix} \begin{bmatrix} x_g \\ \psi_{xg} \end{bmatrix} + \begin{bmatrix} k_i \\ l_c k_i \end{bmatrix} i_x = \begin{bmatrix} k_l & k_{lt} \\ k_{lt} & k_l \end{bmatrix} \begin{bmatrix} x_g \\ \psi_{xg} \end{bmatrix} + \begin{bmatrix} k_i \\ l_c k_i \end{bmatrix} i_x \quad (21)$$

and the sensor outputs are

$$x_s = \begin{bmatrix} 1 & l_s \end{bmatrix} \begin{bmatrix} x_g \\ \psi_{xg} \end{bmatrix} \quad (22)$$

The measured HAMB parameters are shown in Table 2. The measurement is implemented by connecting the HAMB rotor to the short shaft and supporting one side of the shaft by a ball bearing. Table 2 also shows the theoretical values when assuming $B_{b0} = 0.22$ [T]. Although some differences between HAMB1 and HAMB2 or between the x and y directions are observed, the measured values are comparable to the theoretical values.

Table 2 Measured HAMB parameters.

		k_l [N/mm]	k_i [N/A]
HAMB1	x	39.6	21.3
	y	38.2	20.9
HAMB2	x	39.1	25.7
	y	40.9	23.2
Theoretical value		39	24

3.2 Flexible rotor model

For simplicity, the gyro term is ignored and the motion in the vertical direction is considered. The coordinate system of the rotor is defined as shown in Fig. 5.

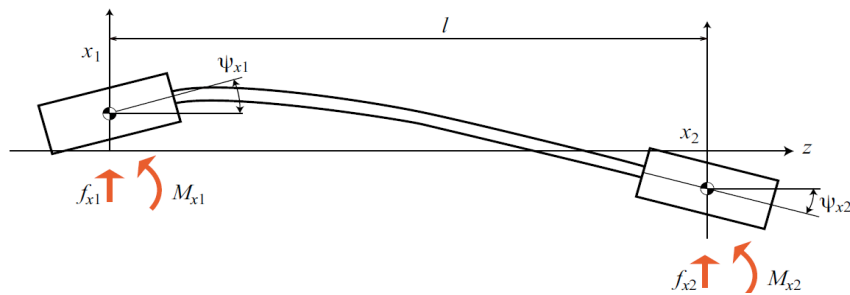


Fig. 5 Flexible rotor. Radial displacements and angles of the HAMB rotors are defined as x_1 , x_2 , ψ_{x1} and ψ_{x2} , respectively. Translational forces and tilt moments applied to the rotors are defined as f_{x1} , f_{x2} , M_{x1} and M_{x2} , respectively.

The force and moment that act on the rotor by the shaft are given as follows (Pilkey, et al., 1989):

$$\mathbf{F}_s = \mathbf{K}_s \mathbf{x} \quad (23)$$

where

$$\mathbf{F}_s = \begin{bmatrix} f_{sx1} \\ M_{sx1} \\ f_{sx2} \\ M_{sx2} \end{bmatrix}, \quad \mathbf{K}_s = -\frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ \psi_{x1} \\ x_2 \\ \psi_{x2} \end{bmatrix} \quad (24)$$

The force and moment by the HAMB are given as

$$\mathbf{F}_b = \mathbf{K}_b \mathbf{x} + \mathbf{K}_i \mathbf{i} \quad (25)$$

where

$$\mathbf{F}_b = \begin{bmatrix} f_{bx1} \\ M_{bx1} \\ f_{bx2} \\ M_{bx2} \end{bmatrix}, \quad \mathbf{K}_b = \begin{bmatrix} k_l & k_{lt} & 0 & 0 \\ k_{lt} & k_t & 0 & 0 \\ 0 & 0 & k_l & k_{lt} \\ 0 & 0 & k_{lt} & k_t \end{bmatrix}, \quad \mathbf{K}_i = \begin{bmatrix} k_i & 0 \\ l_c k_i & 0 \\ 0 & k_i \\ 0 & -l_c k_i \end{bmatrix}, \quad \mathbf{i} = \begin{bmatrix} i_{x1} \\ i_{x2} \end{bmatrix} \quad (26)$$

Then, the equation of motion of the rotor becomes

$$\mathbf{M} \ddot{\mathbf{x}} = (\mathbf{K}_s + \mathbf{K}_b) \mathbf{x} + \mathbf{K}_i \mathbf{i} \quad (27)$$

where

$$\mathbf{M} = \text{diag}(m_r, J_r, m_r, J_r) \quad (28)$$

m_r is the mass of the rotor, and J_r is moment of inertia of the rotor. The equation of state then becomes

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \ddot{\mathbf{x}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{M}^{-1}(\mathbf{K}_s + \mathbf{K}_b) & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \mathbf{K}_i \end{bmatrix} \mathbf{i} \quad (29)$$

and the output equation is

$$\mathbf{y} = \begin{bmatrix} x_{s1} \\ x_{s2} \end{bmatrix} = \begin{bmatrix} 1 & l_s & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -l_s & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix} \quad (30)$$

The eigenvalue analysis (Yasuda, 1993) of the equation of motion is shown in Table 3. The results reveal that flexible modes exist around 45 and 140 Hz.

Table 3 Result of eigenvalue analysis.

	Eigenvalue	Natural freq.[Hz]	Eigenvector
λ_1	-3.3×10^4	-	[0.6604 0.6962 -0.2266 -0.0466]
λ_2	-3.3×10^4	-	[0.036 -1.9183 10.1518 -15.6352]
λ_3	7.6×10^4	44	[0.6602 0.4828 0.8416 -1.4349]
λ_4	7.7×10^5	142	[-0.1055 -2.1213 8.8640 -7.3594]

3.3 Hanging Test

The hanging test is implemented to inspect the natural frequency and flexible mode of the flexible rotor. First, we add vibration to the horizontally suspended rotor and carry out FFT analysis of the outputs of the acceleration sensors installed on both ends of the shaft. In Fig. 6, the acceleration sensor signal is shown for only the HAMB1 side because the signals are almost identical on both sides. Flexible mode frequencies exist near 45 and 140 Hz, presumably because rigid vibration modes exist at quite low frequencies in the no stiffness condition. To verify the vibration mode in practice, a band-pass filter with a filter frequency of 45 or 140 Hz was attached to the output of the acceleration sensor to detect only the flexible mode signal. These results are shown in Fig. 7. Figs. 7(a) and 7(b) show the cases of 45 and 140 Hz, respectively. In the case of 45 Hz, the signals on both sides are in phase, which means that the first flexible mode exists at 45 Hz. In contrast, in the case of 140 Hz, the signals on both sides are in out of phase, meaning that the second flexible mode occurs at 140 Hz. These results indicate the validity of the flexible rotor model in section 3.2.

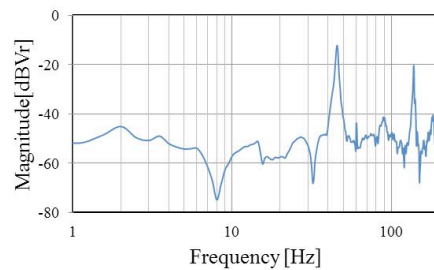


Fig. 6 FFT analysis of acceleration sensor. Resonance frequencies appear at 45 Hz and 140 Hz

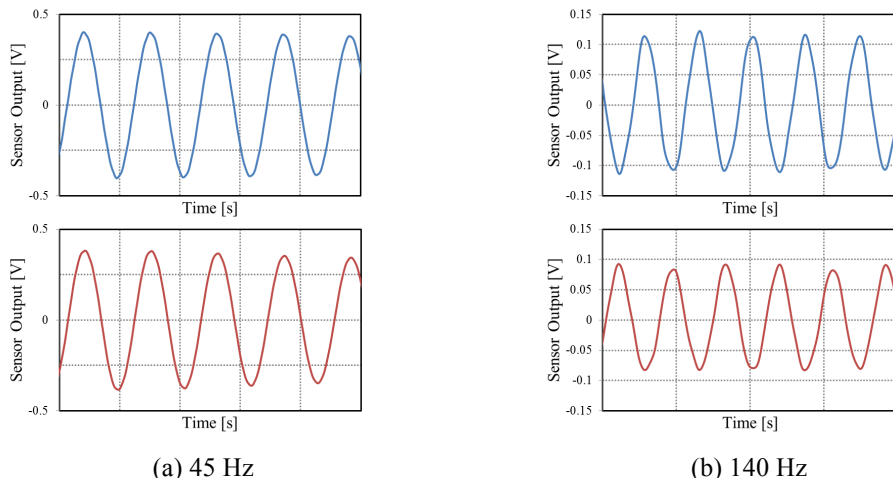


Fig. 7 Acceleration sensor signals after band pass filtering at (a) 45 Hz and (b) 140 Hz. Both shaft ends vibrate in phase at 45 Hz and out of phase at 140 Hz.

3.4 Controller

The central PID controller is used for levitation control and the notch filter is used to prevent destabilization of the flexible mode (Kanemitsu, et al., 2004). A block diagram of the controller is shown in Fig. 8. It is discretized by the Tustin method to perform the control on the DSP. The parameters of controller are shown in Table 4.

Table 4 Parameters of controller.

x y			ψ_x, ψ_y		T_d	1.0×10^{-3}	s
K_p	9000	N/m	300	Nm/rad	Sampling time	0.04×10^{-3}	s
K_d	13	Ns/m	1.5	Nms/rad	ω_c	$2\pi \times 140$	rad/s
K_i	35	50 N/ms	2.0	Nm/rads	ζ	0.01	

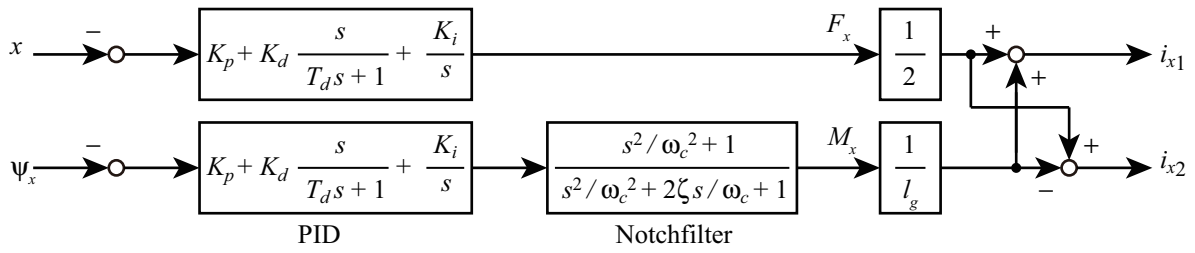
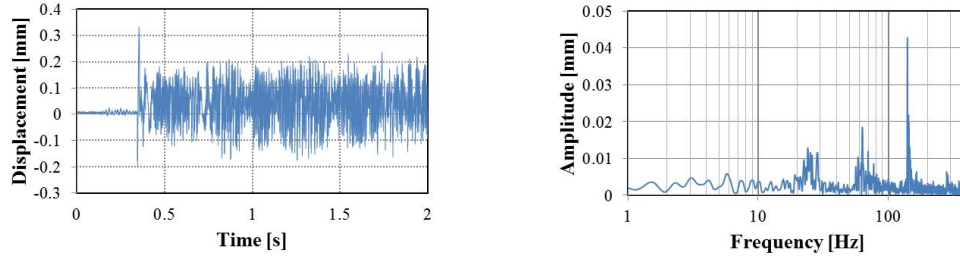
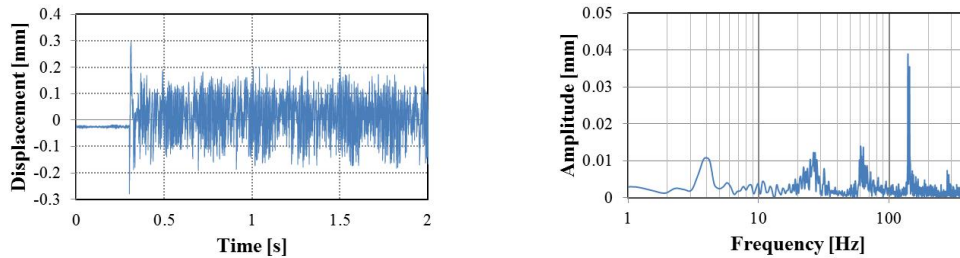


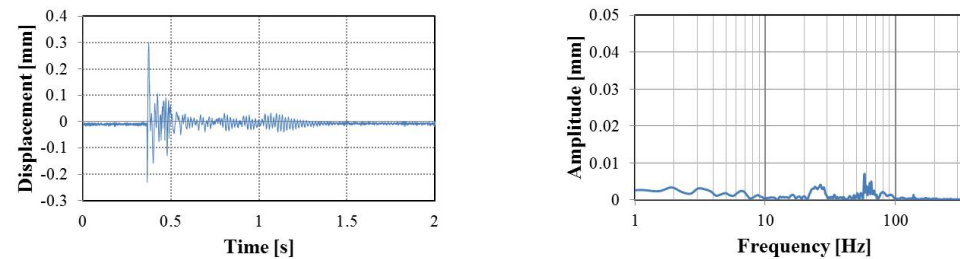
Fig. 8 Block diagram of central PID controller.



(a) Without Notch Filter



(b) With Notch Filter in Displacement



(c) With Notch Filter in Displacement Angle

Fig. 9 Impulse responses and FFT analysis when a notch filter was (a) not applied, (b) applied to the displacement and (c) applied to the angle. A vibration of round 140 Hz is produced in the first two cases and settled in the third.

4. Levitation and Rotation tests

The results of the levitation test are shown in Fig. 9, which reveal the transient and frequency characteristics of the displacement when an impulse disturbance is added to the center of the shaft as a levitating rotor. Figs. 9(a), 9(b) and 9(c) show the cases when the notch filter is not applied, applied to the displacement and to the angle, respectively. The left panels show the time course of the vertical displacement of HAMB1, whereas the right panels show their corresponding amplitude spectra. These figures confirm that the natural frequency is around 140 Hz. The vibration near 140 Hz increased and levitation became unstable when the notch filter was not applied and when it was applied to the displacement. In contrast, applying notch filter to the angle, did not stimulate vibration around 140 Hz, and ultimately, achieved stable levitation. This method was applied to a symmetric rotor in this work. The effectiveness of this method in the case of asymmetric rotors is under investigation.

There is also a peak around 60Hz in the result of FFT analysis of Fig. 9. This peak is a first rigid mode because when the proportional gain K_p of the displacement is changed to higher value this peak moves to higher frequency, on the other hand when the K_p of the angle is changed to higher value this peak does not move.

A rotation test was conducted by connecting the side of the shaft with HAMB2 to a DC motor. The result is shown in Fig. 10. The vertical axis shows the peak-to-peak value of the y-direction displacement of HAMB1 at each rotation speed. The rotor became even more stable when the rotation speed increased to the resonance frequency of 140 Hz, which is about 8400 rpm. Moreover, peak-to-peak value at 3000rpm is large because of influence of first rigid mode around the 60Hz.

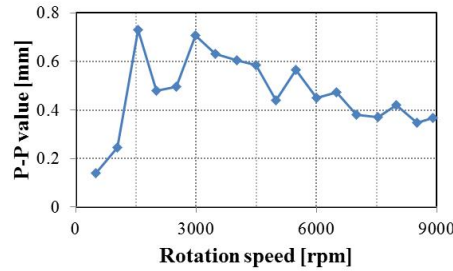


Fig. 10 Peak-to-peak value at each rotation speed.

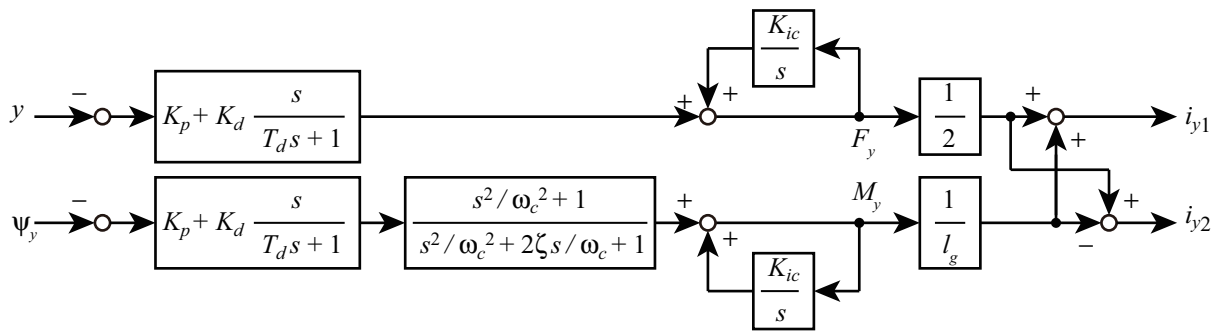


Fig. 11 Block diagram of zero power control.

5. Zero power control

Zero power control regulates the flexible rotor to the position where the force of gravity equals the levitation force by creating an imbalance in the magnetic flux of the permanent magnet. The zero power control system is shown in Fig. 11. The control value after the PD controller of the displacement control and angle control are fed back to their control value through an integrator, respectively (Sun and Oka, 2009). Zero power control is applied to control in both the horizontal and vertical directions. The integrator gains of the displacement control and angle control are 0.012 [1/s] and 0.01 [1/s]. Figure 12 shows the control current values and y-direction displacements in the each case of PID control or zero power control. The upper graphs compare of control currents that control HAMB1 and HAMB2 in vertical direction. The absolute values of the control currents are smaller when zero power control is applied. The lower graphs show the corresponding displacements for each case. The results confirm that the flexible rotor is suspended at a position higher than the center of the magnetic bearing's inner diameter in the case of zero power control. These results indicate that zero power control can perform levitation with low power consumption.

The rotation test was conducted with zero power control. Figure 13 compares the power consumption and peak-to-peak values of HAMB1 in y direction with those of the PID control at each rotation speed. The results confirm that zero power control supports rotation of the flexible rotor with low current value with hardly any change in the amplitude of vibration. The rotation speed may reach 8500 rpm. Zero power control can also stably pass through the second flexible mode frequency. In Fig. 13, a peak appears at 4000 rpm in zero power control and the frequency is higher than that of the result with the PID controller. It is supposed that the bearing stiffness might change in zero power control because of a shift of an equilibrium point. However, the second flexible mode is not affected by the change of stiffness because the frequency of the notch filter is not changed. A P-P value grows big in more than 7000 rpm. However, higher rotation speed was limited by capacity of DC motor, so there is room for examination about it.

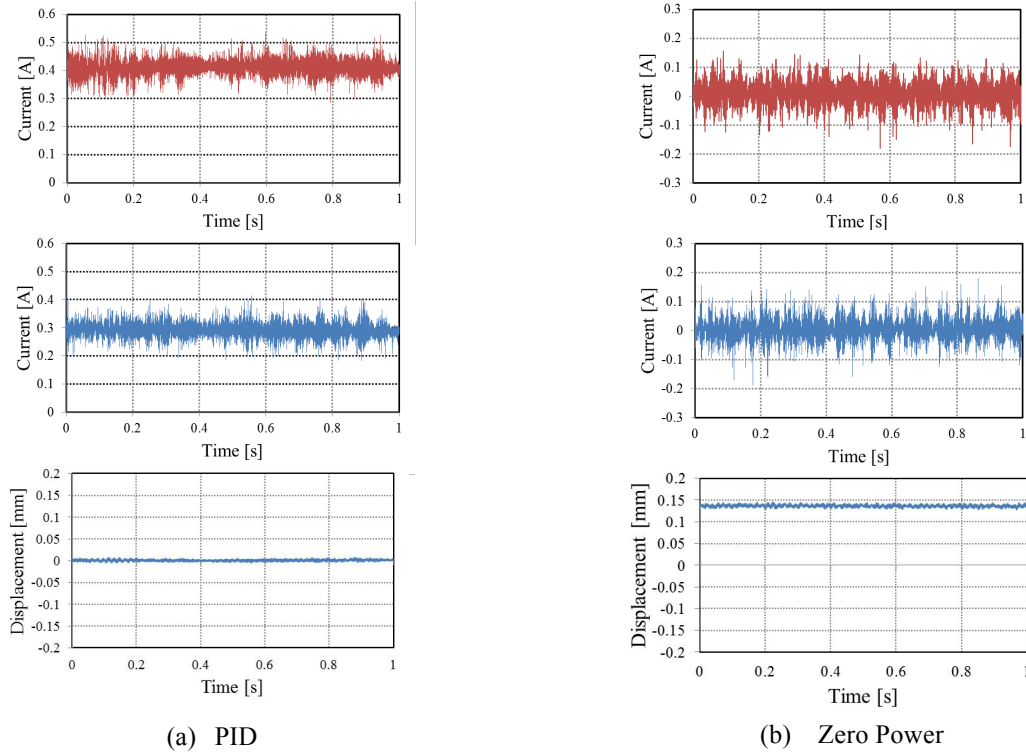


Fig. 12 Comparison of current value and the time course of displacement between (a) PID and (b) Zero power control on levitation test. The current for Zero power control is smaller, and the levitating point higher, compared with those of PID control.

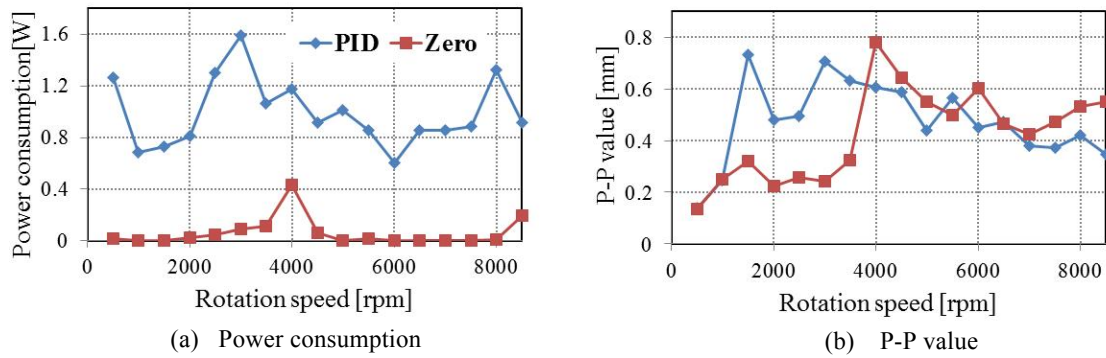


Fig. 13 Comparison of power consumption and P-P value between PID control and Zero power control at each rotation speed. The current for Zero power control is smaller than that for PID control

6. LQR control

Here, LQR control theory is applied to a rigid model that considers the first and second modes. The displacement and the angle of the rigid model are calculated from the sensor values. $\mathbf{Q}$ and $\mathbf{R}$ are weight matrices given for the quadratic performance index J

$$J = \int_0^{\infty} (\mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{u}^T \mathbf{R} \mathbf{u}) dt \quad (31)$$

where $\mathbf{Q} = \text{diag}[1 \ 1 \ 1 \ 1]$, $\mathbf{R} = \text{diag}[1 \ 1]$ in this time.

The control input that minimizes the performance index J is calculated as follows

$$\mathbf{u} = -\mathbf{R}^{-1} \mathbf{B}^T \mathbf{P} \mathbf{x} = -\mathbf{F} \mathbf{x} \quad (32)$$

where $\mathbf{F}$ is the feedback gain and $\mathbf{P}$ is the solution of the Riccati equation

$$\mathbf{P}\mathbf{A} + \mathbf{A}^T\mathbf{P} - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} + \mathbf{Q} = \mathbf{0} \quad (33)$$

The levitation test was conducted by using the obtained feedback gain. Fig. 14 shows the output of a horizontal displace sensor when an impulse disturbance is added and stable levitation with the LQR controller is confirmed.

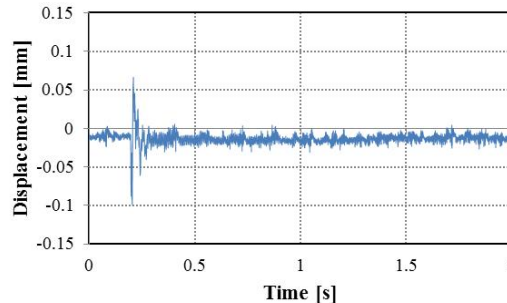


Fig. 14 Impulse response on levitation test. Stable response is confirmed with LQR control.

7. Conclusion

In this paper, levitation control and the rotation test for a flexible rotor were performed by a HAMB, which combines permanent magnets and electrical magnets. First, the HAMB was shown to have negative stiffness not only in the radial direction but also in the angular direction. Next, mathematical models of the rotor and controller were introduced. In experiments, levitation was unstable with only a PID controller because of a flexible mode. Hence, a notch filter was installed to restrain this mode, and stable levitation was achieved. Subsequently, we confirmed that the rotor could operate stably at the second flexible mode frequency in rotation test. Finally, the effectiveness of zero power control for reducing electricity consumption and stable levitation with LQR control were confirmed.

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